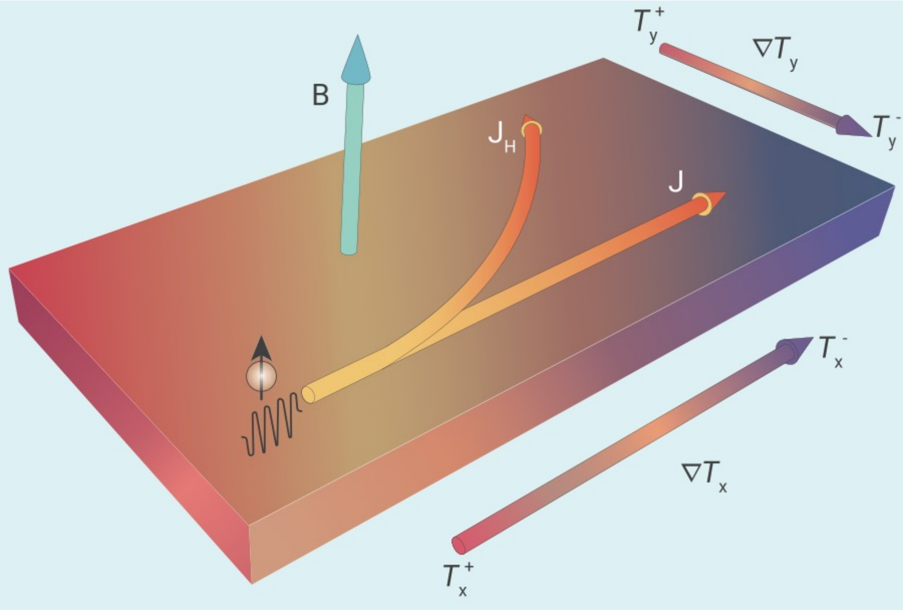
Thermal Hall effect to probe topological properties

Alexandre Pourret

PHELIQS Laboratory CEA Grenoble



Topo School 2024

Summary

- History of the anomalous Hall effect (AHE)

- Berry curvature: Intrinsic AHE
  Example: Skyrmions lattice in EuPtSi (Berry phase in real space)

- The different thermo-electric coefficients

- The Thermal Hall effect (THE)
  Examples: Phonon THE (skew scattering, chiral phonons….)
  Magnon THE

- New perspectives using THE in p-wave superconductors

History of the Anomalous Hall effect

Hall 1879: Ordinary Hall effect of classical physics requires an external magnetic field $\rho_{xy} = R_o B$, where $R_o = 1/(ne)$

Hall 1880: Anomalous Hall effect requires only a magnetisation, ferromagnets display a spontaneous Hall conductivity in the absence of an external magnetic field

The mechanisms responsible for this deflection have been the subject of substantial controversy ever since Hall's experimental work

Several mechanisms are known to be responsible, **spin-orbit interaction** is the main one:
intrinsic part (material's band structure) and **extrinsic process** (scattering events)

Intrinsic spin-orbit coupling effect are intertwined with the topological properties

3 distinct periods in the understanding of the AHE

History of the Anomalous Hall effect

Classic Period (1880-1990)

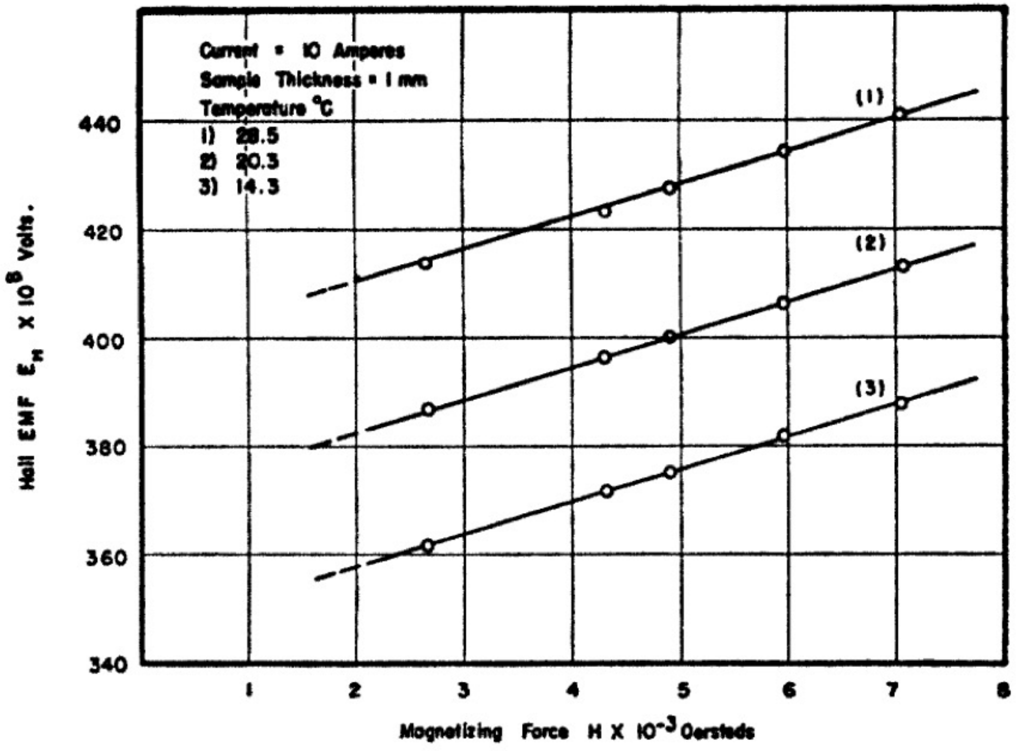
Anomalous Hall effect in transition metals, and on its exact relationship to the magnetisation M.

Empirically : $\rho_{xy} = R_0 B + R_S M$, with the constant R_S referred to as the anomalous Hall coefficient (Pugh & Lippert, 1932).

Frequently $R_s \gg R_0$ (Pugh, et al., 1950)
Spontaneous Hall current indeed exists in the absence of a magnetic field, identification of ferromagnetic order.

Contreversy: Role of impurities ((Karplus & Luttinger, 1954 Vs Smit 1955): intrinsic vs extrinsic

Ferromagnetic system

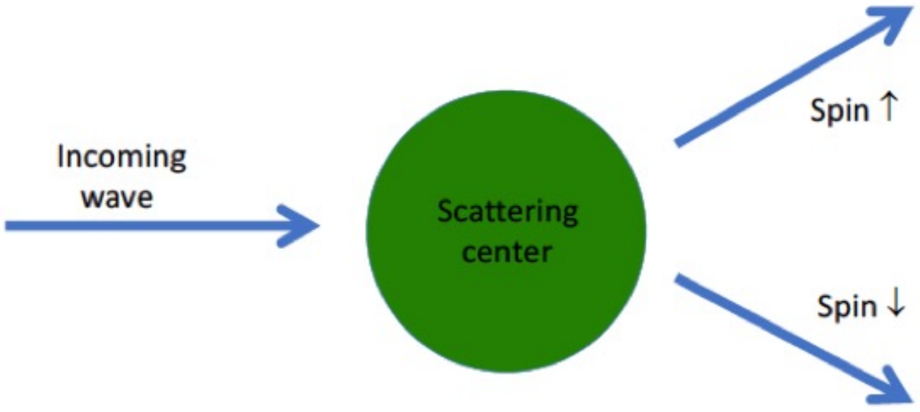


(Pugh & Lippert 1932).

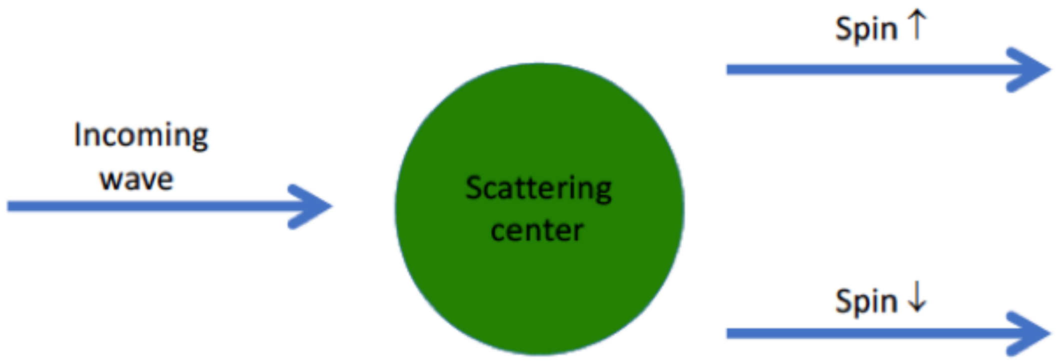
Extrinsic process of the AHE

Skew scattering: Spin-up and spin-down electrons are scattered preferentially in different directions.
(Smit, 1955; Luttinger & Kohn, 1955; Kohn & Luttinger, 1957; Luttinger, 1958; Luttinger & Kohn, 1958; Adams & Blount, 1959)

$$\rho_{xy} \propto \rho_{xx}$$



Side-jump: Spin-up and spin-down electrons undergo a lateral shift in different directions during a scattering process.
(Berger, 1970)



$$\rho_{xy} \propto \rho_{xx}^2$$

History of Anomalous Hall effect

Modern Period (1990-2010)

Intrinsic process, scattering-free contribution of Luttinger and Karplus is related to the Berry curvature of Bloch electrons $\rho_{int} \propto \rho_{xx}^2$

Three resistivity regimes have been identified:

- a high mobility regime : skew scattering dominates because τ is large (Majumdar & Berger, 1973)
- a good metal regime : the dominant contribution is independent of τ (Miyasato, et al., 2007), and may be intrinsic or side-jump or both
- a low-mobility regime : conduction occurs primarily by hopping, where the overall picture remains unclear (Nagaosa, et al., 2010).

History of Anomalous Hall effect

Topological Period (2010-today)

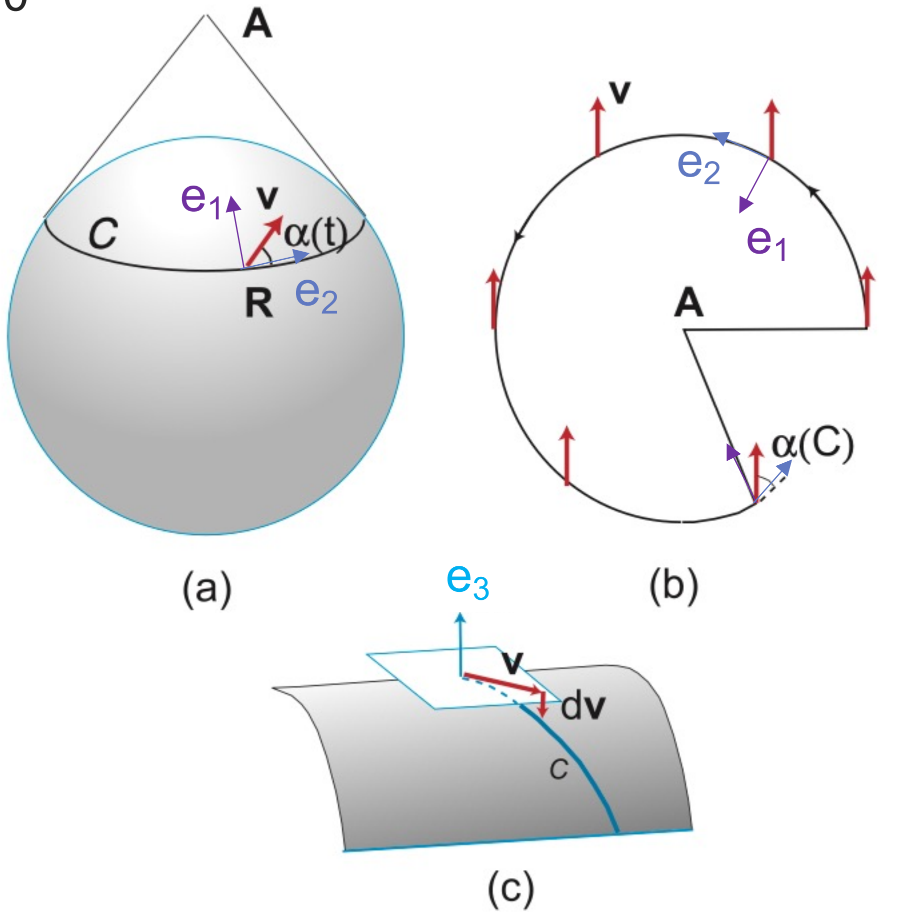
Discovery of many topological materials including topological insulators, Weyl semimetals, transition metal dichalcogenides, and van der Waals heterostructures.

Topological materials have brought with them a new understanding of the AHE
Anomalous Hall effect can be quantised in topological insulators and Weyl semimetals

Geometrical phase

Unit vector v(t) lie in the local tangent plane (part of a cone):

$v \cdot e_3 = 0$



Parallel transport on a sphere:

$e_3 \times dv = 0$ (Levi-Civita transport)

v does not return to its initial direction when the path is completed.

"geometric" angle α(C) that is path dependent (holonomy)

Mesuring α(C):

$v(t) = \cos\alpha(t)\, e_1(t) + \sin\alpha(t)\, e_2(t)$

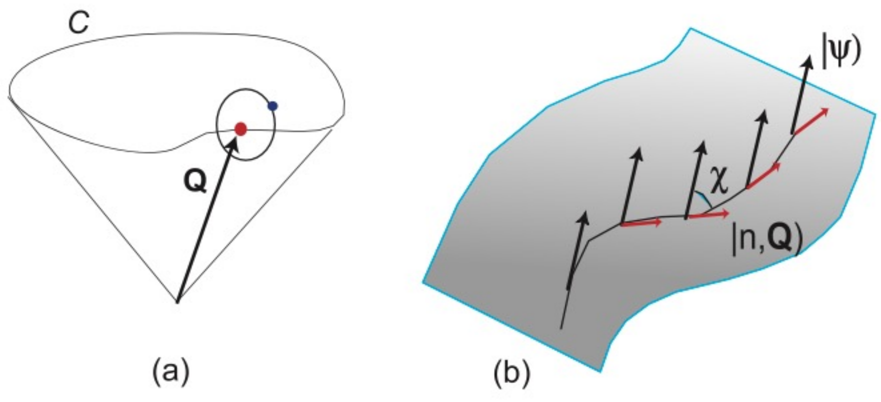
$w = e_3 \times v$

$[v, w, e_3]$ rotates around the normal by α(t) relative to $[e_1, e_2, e_3]$

$e_3 \times dw = 0$

Finally $d\alpha = e_1 \cdot de_2 \equiv \omega_{21}$

Berry phase for an electron



Electron constrained to same eigenstate
as nuclear coordinate Q is taken around a closed path C
will acquire a phase :

$$|\psi\rangle = |n, \mathbf{Q}\rangle e^{-i\chi}$$

Parallel transport:

$$\langle n, \mathbf{Q}|\delta|\psi\rangle = 0$$

Berry phase $\chi = -\langle n, \mathbf{Q}|i\delta|n, \mathbf{Q}\rangle$

On completing the path C, the total Berry phase χ(C) is the line integral

$$\chi(C) = -\oint_C d\mathbf{Q} \cdot \langle n, \mathbf{Q}|i\nabla_{\mathbf{Q}}|n, \mathbf{Q}\rangle$$

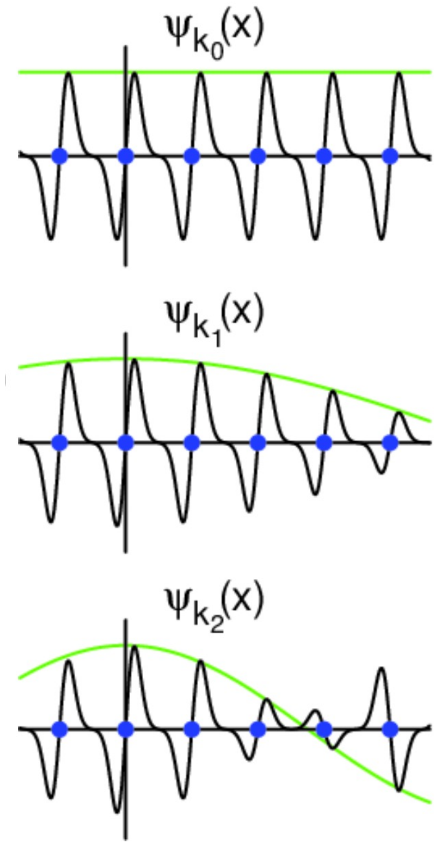
Physically, as Q changes, the electronic ket |ψ⟩ stays "parallel" to its initial direction while the local reference ket |n, Q⟩ rotates relative to it.

The Berry phase is the phase angle between them.
Vector potential and effective magnetic field representation:

$$\mathbf{A}(\mathbf{Q}) = \langle n, \mathbf{Q}|i\nabla_{\mathbf{Q}}|n, \mathbf{Q}\rangle$$

$$\mathbf{B}(\mathbf{Q}) = \nabla_{\mathbf{Q}} \times \mathbf{A}(\mathbf{Q})$$ Berry curvature

Berry phase for Bloch electron



Bloch States :

$$\psi_{n\mathbf{k}}(\mathbf{r}) = \frac{1}{\sqrt{N}} e^{i\mathbf{k}\cdot\mathbf{r}} u_{n\mathbf{k}}(\mathbf{r}) = \langle \mathbf{r} | n\mathbf{k} \rangle$$

band index n and wave vector k

The electron wave vector will acquire a Berry phase

Berry phase $$\chi(\mathbf{k}) = -\int_C^{\mathbf{k}} d\mathbf{k}' \cdot \mathbf{X}(\mathbf{k}')$$

Berry potential vector $$\mathbf{X}(\mathbf{k}) = \int_{cell} d^3r\, u^*_{n\mathbf{k}}(\mathbf{r}) i \nabla_{\mathbf{k}} u_{n\mathbf{k}}(\mathbf{r})$$

Berry curvature $$\mathbf{\Omega}(\mathbf{k}) = \nabla_{\mathbf{k}} \times \mathbf{X}(\mathbf{k})$$

$$\begin{aligned} \hbar\dot{\mathbf{k}} &= e\mathbf{E} + e\mathbf{v} \times \mathbf{B} \\ \hbar\mathbf{v} &= \nabla\epsilon_n - e\mathbf{E} \times \mathbf{\Omega}. \end{aligned}$$

Anomalous velocity

Quasiparticles can gain transverse **velocity from non-zero Berry curvature**

Highly similar to the electrons gaining transverse velocity due to the Lorentz force

R. Karplus and J. M. Luttinger, Phys. Rev. **95**, 1154 (1954).

Charge and Heat Transport coefficients

$$J^e = \sigma E - \alpha \nabla T$$

$$J^Q = \beta E - \kappa \nabla T$$

Fourier's law and Ohm's llaw are entangled through thermoelectric phenomena

where α is the **thermoelectric conductivity**, β is the **electro-thermal conductivity**. Both quantities are linked through the Kelvin relation: β = αT .

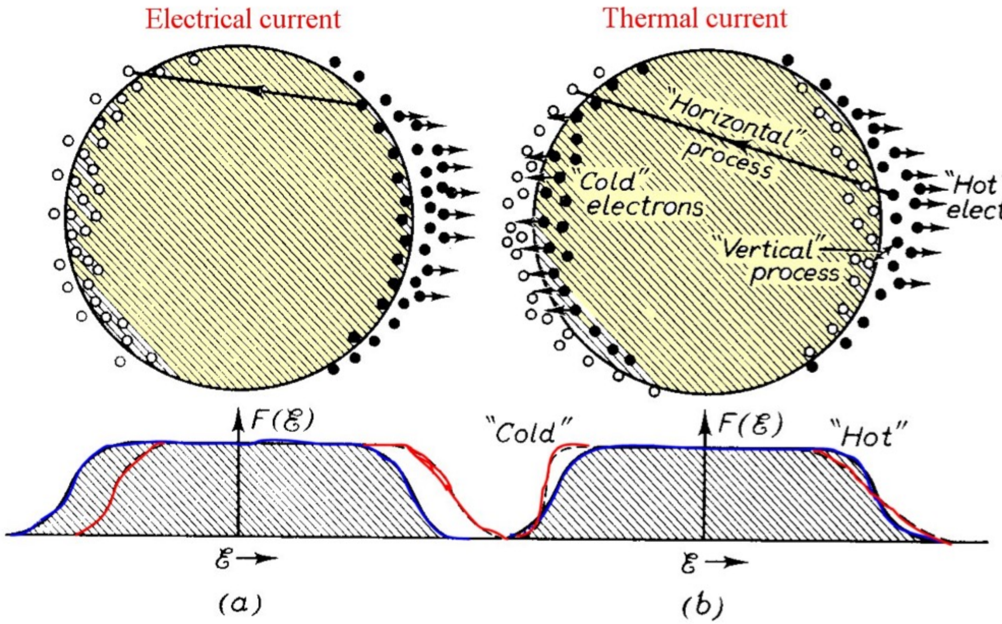
Boltzman approach:

Electrical current flow $$J^e = \int e\vec{v}_k f_k d\vec{k}$$

Heat flux $$J^Q = \int \vec{v}_k(\epsilon_k - \mu) f_k d\vec{k}$$

Fermi-Dirac probability distribution function $$f^0 = \frac{1}{e^{\frac{\epsilon_k - \mu}{k_B T}} + 1}$$

Time differential $$\frac{df_k(\vec{r})}{dt} = -\frac{f_k(\vec{r}) - f^0}{\tau}$$



Charge and Heat Transport coefficients

Charge current

$$J^e = -\int e\vec{v}_k \tau \vec{v}_k \left(\frac{\partial f^0}{\partial \epsilon_k} e\vec{E} + \frac{\partial f^0}{\partial \epsilon_k}\frac{\epsilon_k - \mu}{T}(-\vec{\nabla}T)\right)d\vec{k}$$

Heat current

$$J^Q = -\int \vec{v}_k(\epsilon_k - \mu)\tau \vec{v}_k \left(\frac{\partial f^0}{\partial \epsilon_k} e\vec{E} + \frac{\partial f^0}{\partial \epsilon_k}\frac{\epsilon_k - \mu}{T}(-\vec{\nabla}T)\right)d\vec{k}$$

Electrical conductivity $$\sigma = -e^2 \int v_k^2 \tau \frac{\partial f^0}{\partial \epsilon_k} d\vec{k}$$ Electrons at the Fermi level

Thermoelectric tensor

$$\alpha = -ek_B \int v_k^2 \tau \frac{\partial f^0}{\partial \epsilon_k}\frac{\epsilon_k - \mu}{k_B T} d\vec{k}$$

Electrons just below and above the Fermi level

Thermal conductivity tensor

$$\kappa = -k_B^2 T \int v_k^2 \tau \frac{\partial f^0}{\partial \epsilon_k}\left(\frac{\epsilon_k - \mu}{k_B T}\right)^2 d\vec{k}$$

Charge and Heat Transport coefficients

$$\sigma \approx e^2 \cdot \frac{1}{3} v_k^2 \tau(k) N(\epsilon) = \frac{ne^2\tau}{m}$$

$$\kappa \approx \frac{\pi^2}{3} k_B^2 T \cdot \frac{1}{3} v_k^2 \tau(k) N(\epsilon) = \frac{1}{3} Cvl$$

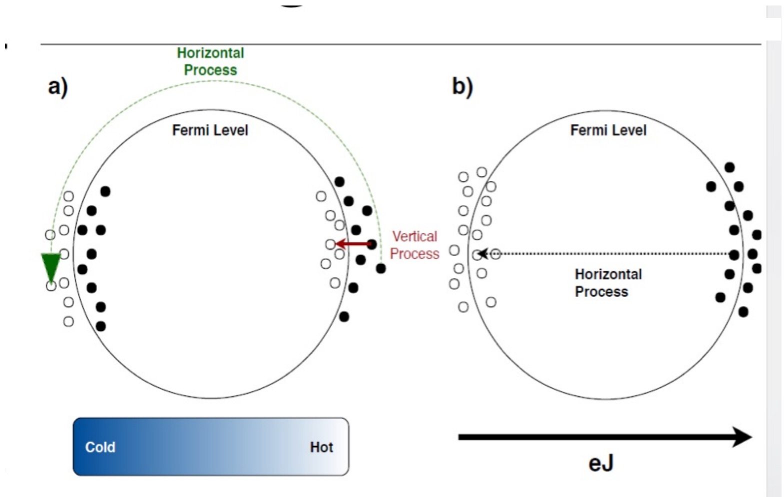
Sommerfeld expansion

Wiedemann-Franz law

$$\frac{\kappa}{\sigma} = \frac{\pi^2}{3} \frac{k_B^2}{e^2} T = L_0 T$$

Lorentz number $L_O = 2.4453 \times 10^{-8} W \Omega K^{-2}$

Only true in the zero temperature limit: elastic scattering is the dominant one.
At finite temperature, inelastic scattering give rise to the deviation of the WF law



Vertical (small-angle) scattering efficiently decays heat current but not charge current

Anomalous Hall effect

$$\mathbf{J} = e\sum_{\mathbf{k},s} \mathbf{v}_{\mathbf{k}}[f^0_{\mathbf{k}} + g_{\mathbf{k}}],$$

the unperturbed fonction $f^0_{\mathbf{k}}$

$$g_{\mathbf{k}} = e\mathbf{v}_{\mathbf{k}}\tau \cdot \mathbf{E}(-\frac{\partial f^0}{\partial \epsilon})$$

+

anomalous velocity

$$\mathbf{J}_H = \frac{e^2}{\hbar}\mathbf{E} \times \sum_{\mathbf{k},\mathbf{s}} \mathbf{\Omega}(\mathbf{k}) f^0_{\mathbf{k}}$$

Anomalous Hall effect will come from summing up all the **velocities from all the occupied states**:

$$\sigma_{xy} = \frac{e^2}{\hbar}\int \frac{d^dk}{(2\pi)^d} f(\epsilon_k)\Omega_{k_x,k_y}$$

Fermi sea

Thus even in the absence of a magnetic field you can get a Hall effect

$\rho_{xy} = \sigma_{xy}\rho^2$ is the quantity that is measured $\rho \sim (n\tau)^{-1}$

$\rho_{xy\ \mathrm{Intrinsic}}$ scales like ρ^2.

$$\sigma_{xy} = \frac{\rho_{xy}}{\rho_{xx}^2 + \rho_{xy}^2}$$

By contrast, skew-scattering theories predict $\sigma_{xy} \sim (n\tau)$ that

$\rho_{xy\ \mathrm{SKEW}}$ is linear in ρ.

Symmetries breaking

What materials have non-zero Ω and hence a significant Anomalous Hall effect?

Ω = 0 if the system has both time reversal and inversion symmetry.
Time reversal takes $v \to -v$, $E \to E$ and $k \to -k$.
$\Omega(-k) = -\Omega(k)$

Inversion symmetry takes $v \to -v$, $E \to -E$ and $k \to -k$.
$\Omega(-k) = \Omega(k)$.

The only consistent way to have both symmetry is to have Ω = 0.

Time reversal symmetry is broken in ferromagnets, antiferromagnets, Weyl Semi Metals

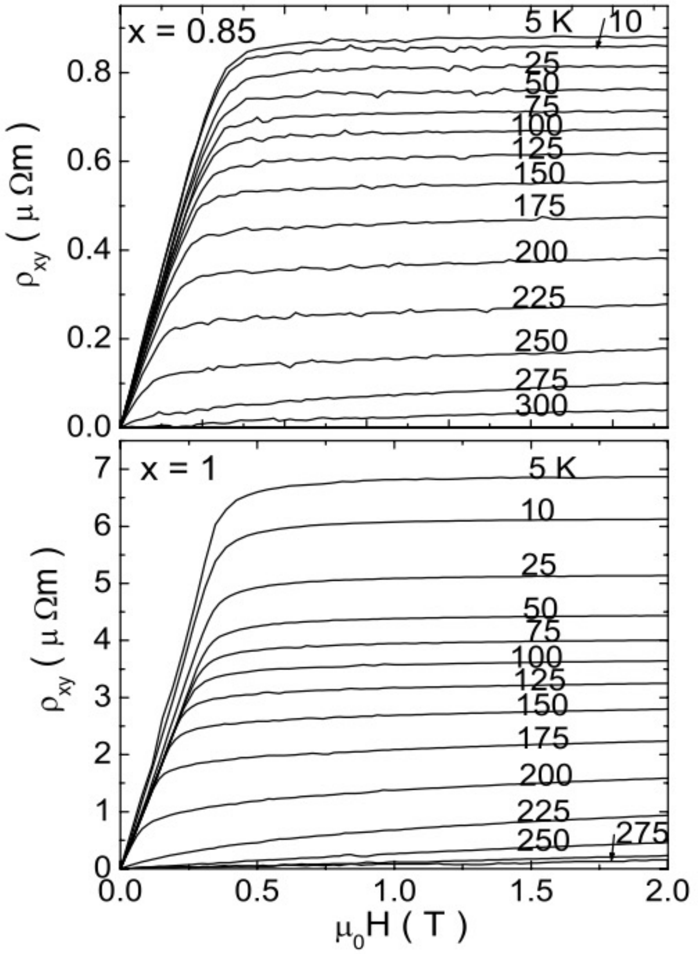
Ferromagnets : M breaks time-reversal symmetry for the spins.

This symmetry-breaking communicated to the charge currents via spin-orbit coupling.

$J_H = 2ne2\lambda E \times S$ that is linear in the carrier density n and M, independent of τ

Spinel Ferromagnet

$CuCr_2Se_{4-x}Br_x$: metal with a Curie temperature $T_C \sim 400$ K (for x = 0)



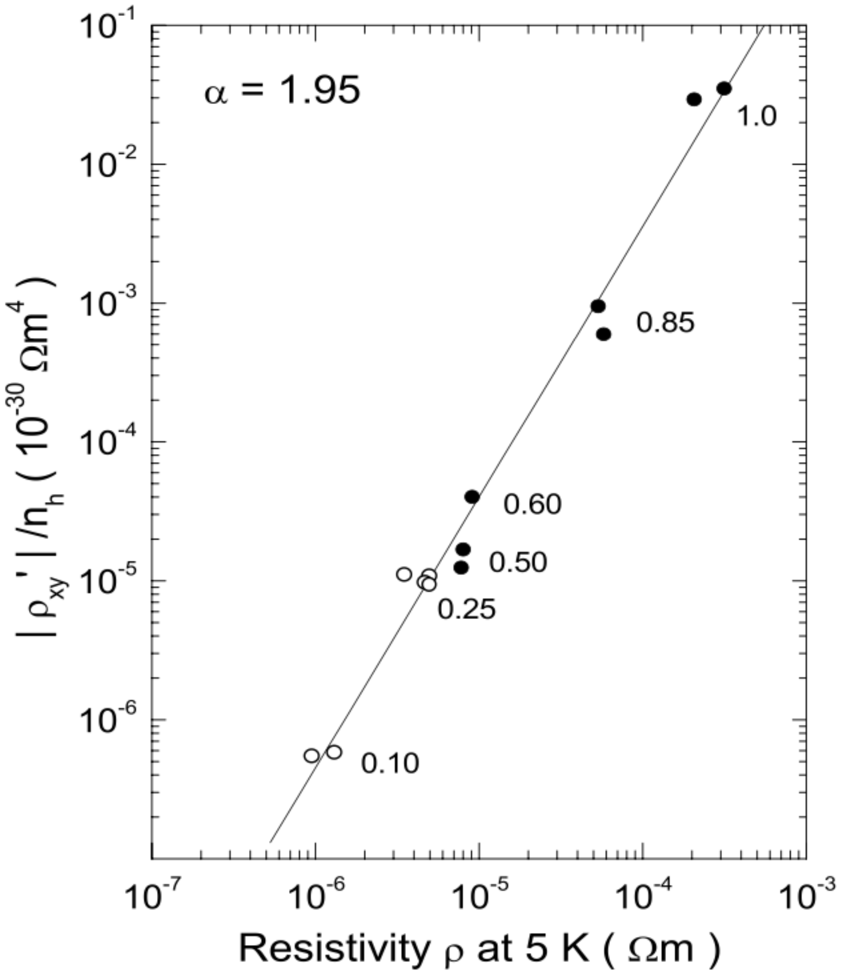
N. P. Ong *et al.* Science (2004)

$\rho_{xy\ \mathrm{Intrinsic}}/n$ scales like ρ^2

n is the hole density

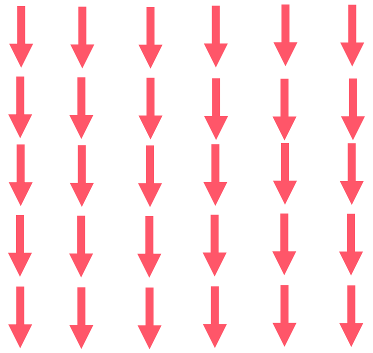
$\boldsymbol{\alpha}$ = 1.95

➢ AHE is independent of $\boldsymbol{\tau}$

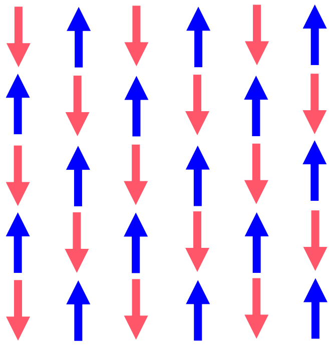


Skyrmions in solid state physics

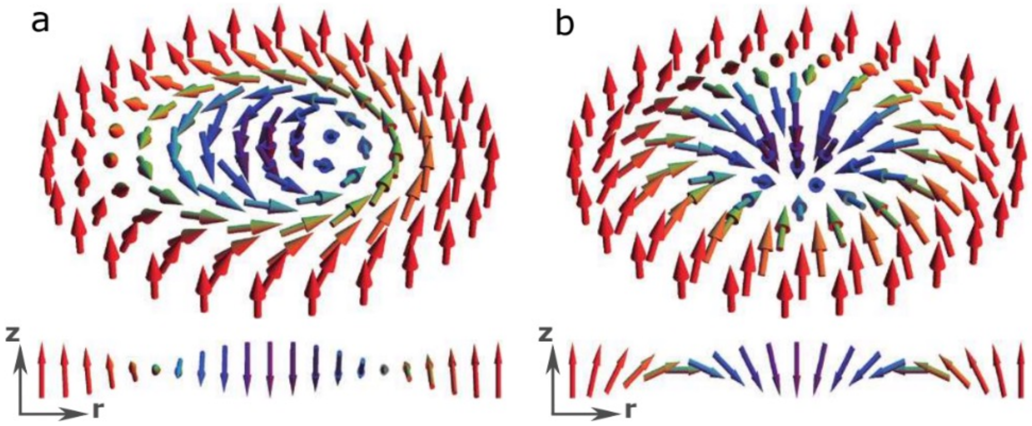
ferromagnetic
order



antiferromagnetic
order



skyrmions



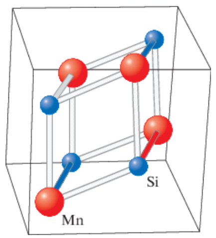
Bloch

Néel

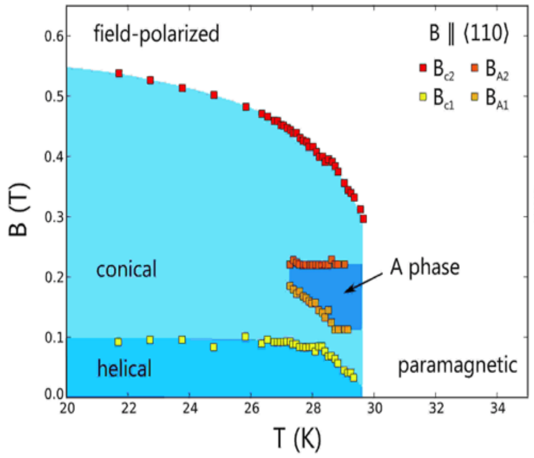
Skyrmion:
- magnetic spin structure in which the center magnetization is opposite to its boundary
- non-trivial real space topology of nano-whirls
- theoretical predictions already in late 80th (Bogdanov and Yablonskii, JETP 1989)
- discovered in bulk materials MnSi (2009)

where to look for skyrmions

systems with broken inversion symmetry (chiral systems)



MnSi, cubic B20



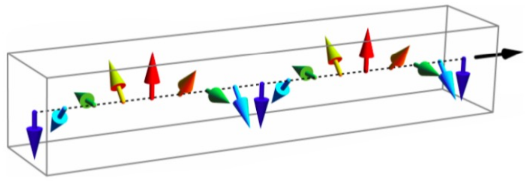
Muhlbauer et Al. , Science, 323, 915-919 (2009)

- exchange interaction with spin orbit

$$H_{ex} = -\frac{1}{2}\sum_{ij} J_{i,j}\, \mathbf{s}_i \cdot \mathbf{s}_j .$$

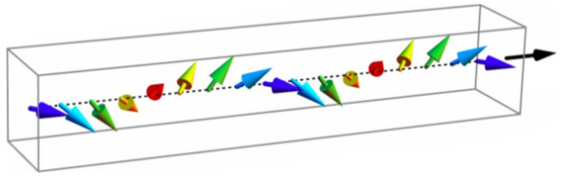
- Dzyaloshinsky – Moriya interaction

$$H_{DM} = \sum_{ij} \boldsymbol{D}_{ij} \cdot \boldsymbol{S}_i \times \boldsymbol{S}_j$$



helical modulation period: $\lambda = 2\pi/|\boldsymbol{q}_H|$ (180 Å)

$$\boldsymbol{m}(\boldsymbol{r}) = \cos(\boldsymbol{q}_H \cdot \boldsymbol{r}) \cdot \boldsymbol{n}_1 + \sin(\boldsymbol{q}_H \cdot \boldsymbol{r}) \cdot \boldsymbol{n}_2$$



conical modulation under magnetic field

- crystal field anisotropy
- magnetic frustration
- dipole – dipole interaction stabilizes large objects

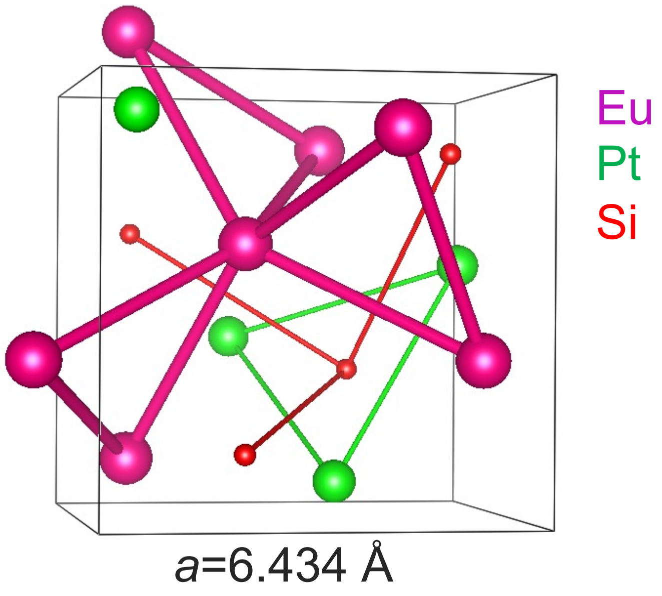
$$H_{dd,ij} = -\frac{\mu_0}{4\pi}\left(3\frac{(\mathbf{s}_i \cdot \boldsymbol{r}_{ij})(\mathbf{s}_j \cdot \boldsymbol{r}_{ij})}{r_{ij}^5} - \frac{\mathbf{s}_i \cdot \mathbf{s}_j}{r_{ij}^3}\right).$$

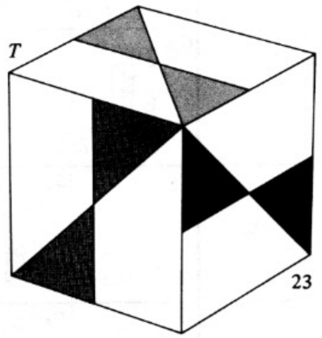
EuPtSi

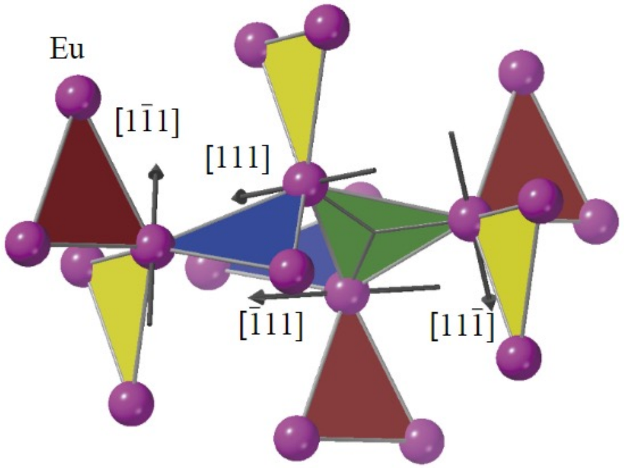
- Rare-earth *4f* ^{7}
- Divalent Eu^{2+} $(L=0,\ S=J=7/2)$
- Large localized moments

- Cubic chiral structure of Ullmannite-type (NiSbS – type) space group $P2_13$ (No. 198) T^4 (like MnSi)
- crystallographic point group 23 (*T* in Schoenflies notation)
- **lack of inversion and mirror symmetry**
- three-fold rotation axis along [111]
- two-fold rotation axis along main axes [100]

- trillium lattice $\rightarrow$ magnetic frustration



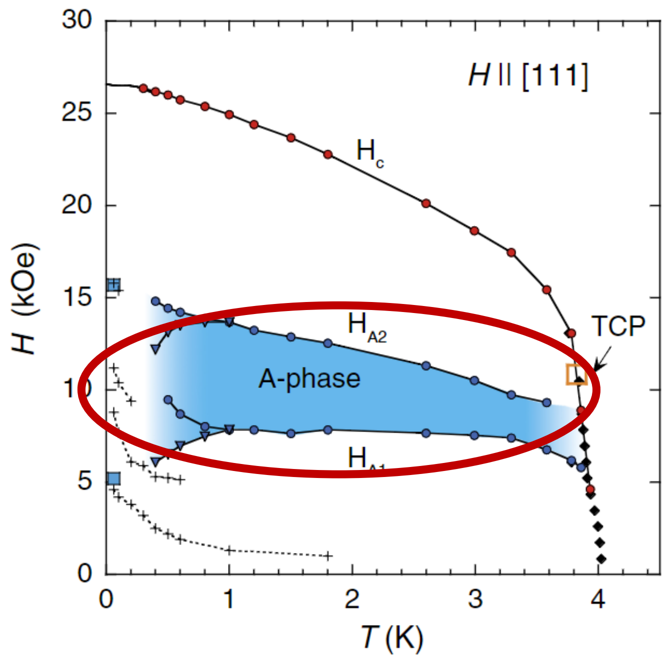




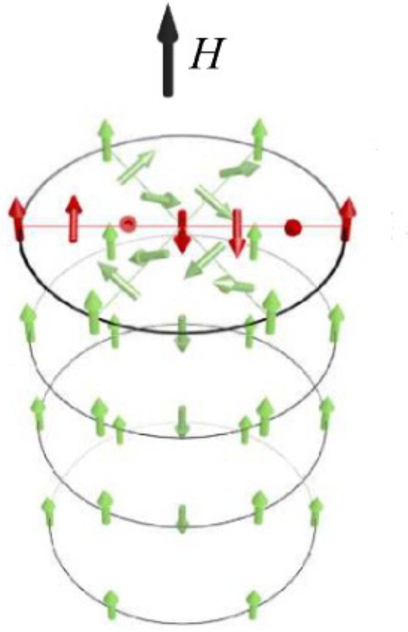
EuPtSi

A-phase in EuPtSi *H* || [111] discovered in 2018 (Kakihana et *al.*)

- From 0.75 to 1.4 T and 0.45 to 4 K → much larger than in MnSi
- topological Hall effect (THE)
- triple-*q* order indicating a Skyrmion Lattice
- small skyrmion size → 10 times smaller than in MnSi



Sakakibara et al. JPSJ **90**, 064701 (2021)

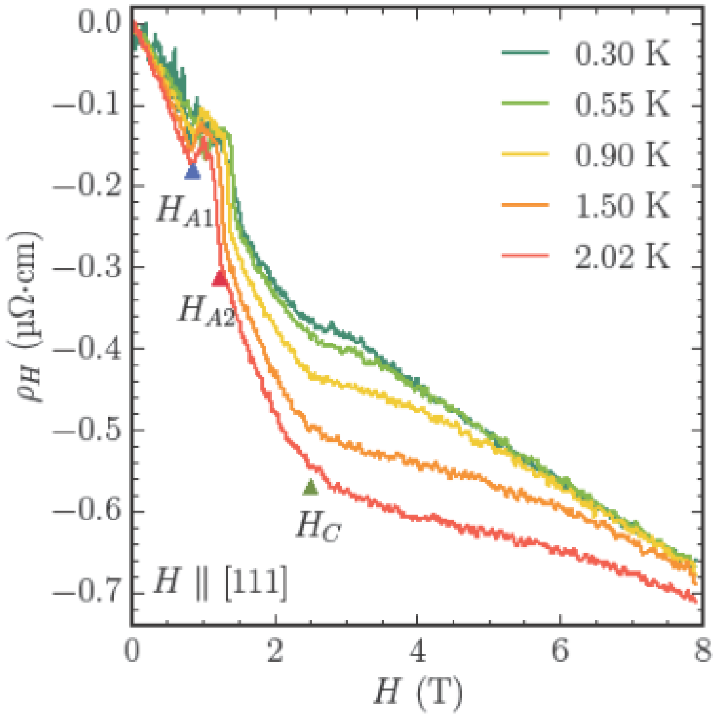


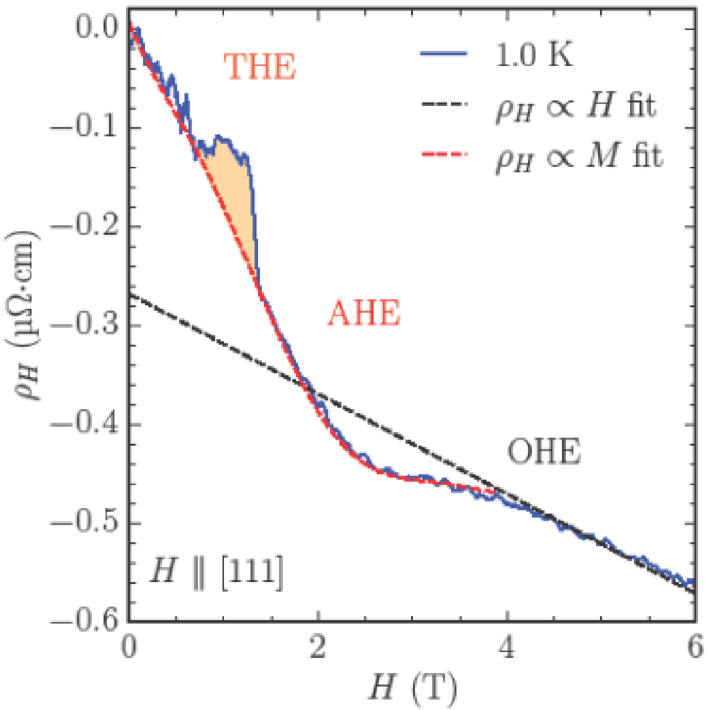
$$q_1 = (-0.29, 0.09, 0.20)$$
$$q_2 = (-0.20, 0.29, -0.09)$$
$$q_3 = (0.09, 0.20, -0.29)$$

$$\lambda = \frac{2\pi}{|\boldsymbol{q}|} = 18\ \text{Å}$$

H || [111] – Topological Hall effect

Topological Hall effect in the A-phase



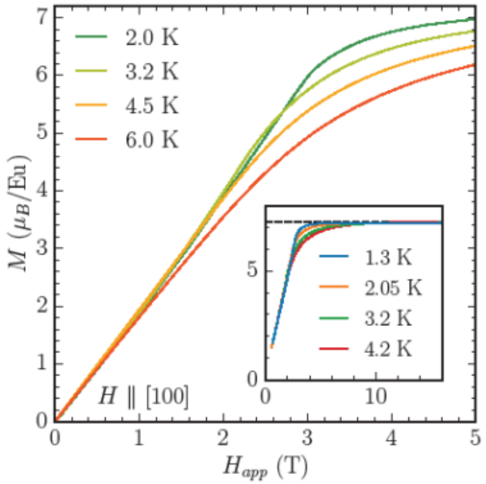


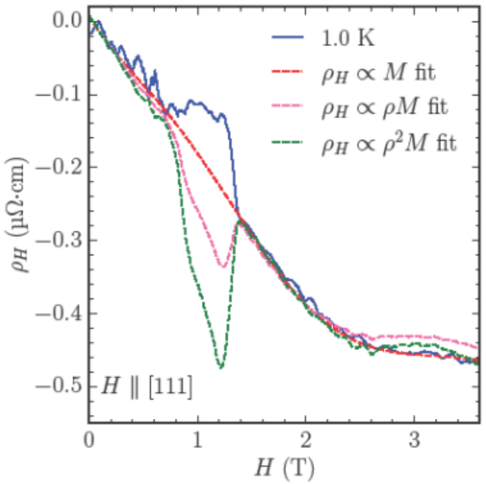
$$\rho_H = \rho_H^{OHE} + \rho_H^{AHE} + \rho_H^{THE}$$

$\rho_H^{OHE} \propto H$ linear in the conical phase

$\rho_H^{AHE} \propto M$ linear in the conical phase

ρ_H^{THE}: additional transverse motion of e- induced by the spin texture



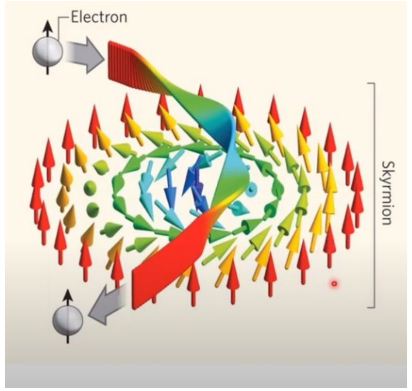


Additional contribution not explained by skew-scattering

H || [111] – Topological Hall effect

Topological Hall effect induced by the spin texture of SkL :

- conduction electrons coupled to non-coplanar spin textures acquire a Berry phase in real space $\rightarrow$ emergent magnetic field B_{em} opposing the applied field
- similar to Lorentz force on the motion of the electron



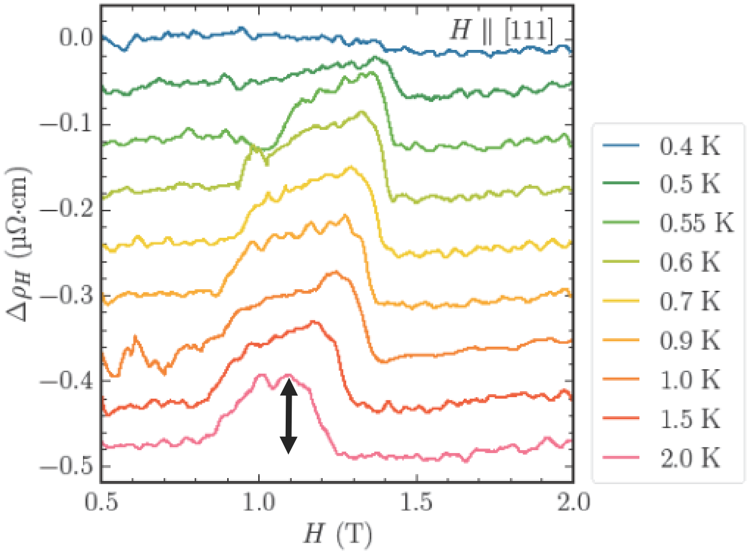
$$B_{em} = -\frac{h}{e}\left(\frac{\sqrt{3}}{2\ \lambda^2}\right) \approx -1105\ \text{T} \qquad \lambda = 18\ \text{Å}$$

$$\rho_H^{THE} \propto P\ R_0 B_{em}$$

- $P = \frac{\mu_{sp}}{\mu_{sat}} \approx 0.1$ Polarisation P arises from the majority and minority-spin carriers, which collect Berry phases of opposite sign.

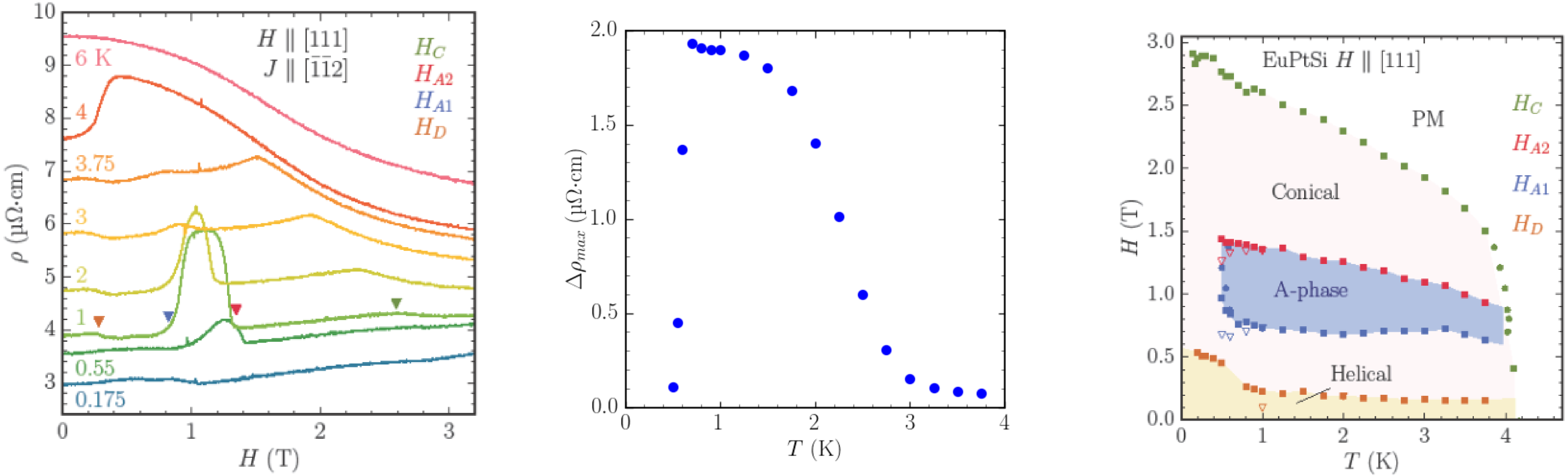
- R_0 the ordinary Hall constant

MnSi: $B_{em} \approx -13.15\ \text{T}$ $\qquad \rho_H^{THE} \approx 5\ \text{n}\Omega \cdot \text{cm}$



EuPtSi:
small skyrmion size → high skyrmion density → large emergent field → large THE $\quad \rho_H^{THE} \approx 0.1\ \mu\Omega \cdot \text{cm}$

H || [111] – Field dependence of resistivity



- large enhancement of ρ in the SkL phase: scattering of conduction electrons on the dense SkL
- rapid decrease of $\Delta\rho_{max}$ below 0.7 K
- Strong hysteresis below 1 K in

From Anomalous Hall effect to Thermal Hall effect

Berry curvature is intricate to the wavefunctions of the energy bands, not with particular quasiparticles:

Electrons, as well as charge-neutral excitations such as magnons and phonons

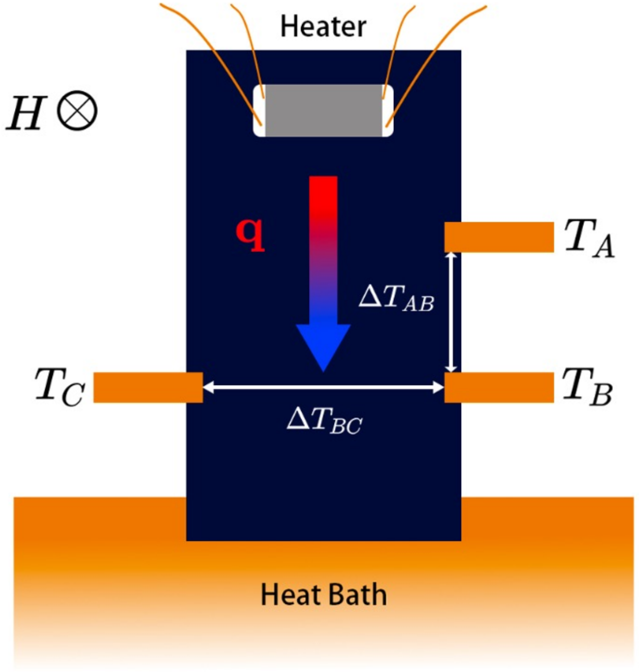
For magnons and phonons: a thermal Hall effect (THE)

For electrons : THE is simply caused by Lorentz force, just the same as the Hall effect

THE occurs even in insulators and that is the attractive and interesting point

It is particularly helpful in unveiling the exotic properties of quantum materials

Thermal Hall Effect Measurement



Thermal conductivity-resistivity

$$\kappa_{xx} = \frac{W_{xx}}{W_{xx}^2 + W_{xy}^2} \qquad W_{xx} = \frac{-\nabla T_x}{J^Q}$$

$$\kappa_{xy} = \frac{-W_{xy}}{W_{xx}^2 + W_{xy}^2} \qquad W_{xy} = \frac{-\nabla T_y}{J^Q}.$$

In most cases, $W_{xx} \gg W_{xy}$, so the $\kappa_{xx} \approx -\frac{J^Q}{\nabla T_x}$ and $\kappa_{xy} \approx -\kappa_{xx}\frac{\nabla T_y}{\nabla T_x}$

INTRINSIC THE

For electrons:

$$\kappa_{\mathrm{H}} = -\frac{1}{e^2\hbar T}\int(\epsilon-\mu)^2\frac{\partial f(\epsilon)}{\partial\epsilon}\boldsymbol{\sigma}(\epsilon)\mathrm{d}\epsilon$$

Fermi-Dirac distribution

$$f(\epsilon) = 1/(e^{(\epsilon-\mu)/(k_B T)} + 1)$$

$$\boldsymbol{\sigma}(\epsilon) = -\frac{e^2}{\hbar V}\sum_{(\varepsilon_n \boldsymbol{q})\leqslant\epsilon}\boldsymbol{\Omega}_n(\boldsymbol{q})$$

Intrinsic AHE

For phonons:

$$\kappa_{\mathrm{H}} = \frac{1}{2V\hbar T}\sum_{\boldsymbol{q},n=1,2nd}\left[\int_{\varepsilon_n(\boldsymbol{q})}^{\infty}\epsilon^2\frac{\partial f}{\partial\epsilon}\mathrm{d}\epsilon\right]\boldsymbol{\Omega}_n(\boldsymbol{q})$$

Bose Einstein distribution

$$f(\epsilon) = 1/(e^{\epsilon/(k_B T)} - 1).$$

The corresponding anomalous velocity of the phonon is proportional to $\Omega\times\nabla T$

For magnons :

$$\kappa^{xy} = -\frac{k_{\mathrm{B}}^2 T}{\hbar V}\sum_{n,\boldsymbol{k}} c_2(\rho_n)\Omega_{n,z}(\boldsymbol{k}).$$

$$\rho(\varepsilon) = (e^{(\varepsilon-\mu)/k_{\mathrm{B}}T} - 1)^{-1},$$

Examples of THE

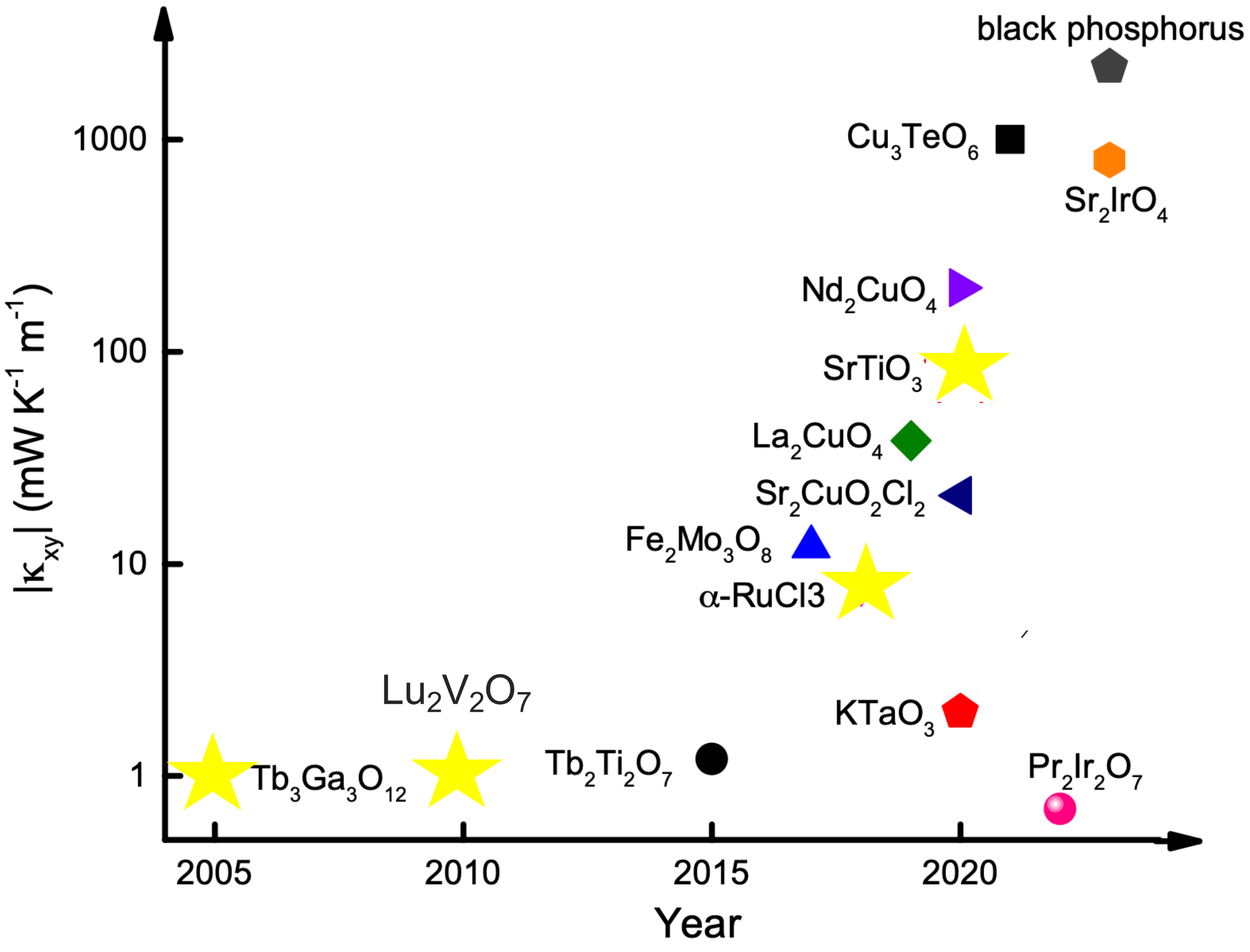
	Material	$\kappa_L(H,T)$ (W/K/m)	$\kappa_H(H,T)$ (mW/K/m)	H (T)	T (K)	Reference(s)
Pyrochlore materials	$Tb_2Ti_2O_7$	0.27	1.2	12	15*	[Hirschberger et al., 2015b]
	$(Tb_{.3}Y_{.7})_2Ti_2O_7$	≈ 0.5	3.0	12	11*	[Hirokane et al., 2019]
	$Y_2Ti_2O_7$	18	0	8.0	15*	[Hirschberger et al., 2015b, Li et al., 2013]
	$Lu_2V_2O_7$	0.7	≈ 1	0.1	50	[Onose et al., 2010, Ideue et al., 2012]
	$Ho_2V_2O_7$	1.0	≈ 0.5	0.1	50	[Ideue et al., 2012]
	$In_2Mn_2O_7$	3.1	≈ -2	0.1	102	[Ideue et al., 2012]
	See also [Kolland et al., 2012, Toews et al., 2013] for studies of κ_L in several other compounds.					
Perovskites	$Fe_2Mo_3O_8$	5.0	27	10	45	[Ideue et al., 2017]
	$(Zn_{1/8}Fe_{7/8})_2Mo_3O_8$	9.0	30	0.1	30	[Ideue et al., 2017]
	$SrTiO_3$	36	80	12	20	[Martelli et al., 2018, Li et al., 2020a]
	$KTaO_3$	32	2	12	30	[Martelli et al., 2018, Li et al., 2020a]
	$BiMnO_3$	≈ 2.7	≈ -0.7	0.1	48	[Ideue et al., 2012]
	$YTiO_3$	≈ 2.7	0	0.1	48	[Ideue et al., 2012]
	La_2TiMnO_6	≈ 0.4	0	0.1	48	[Ideue et al., 2012]
	$Ba_3CuSb_2O_9$	0.8	-0.08	15	50	[Sugii et al., 2017]
Others	VI_3	4.0	≈ 10	>0.1	20	[Zhang et al., 2021a]
	Cu_3TeO_6	$3.3 \cdot 10^2$	$-1.1 \cdot 10^3$	15	20	[Chen et al., 2021]

Examples of THE

	Material	$\kappa_L(H,T)$ (W/K/m)	$\kappa_H(H,T)$ (mW/K/m)	H (T)	T (K)	Reference(s)
Rare-earth garnets	$Tb_3Ga_5O_{12}$	0.19	$2.0 \cdot 10^{-2}$	3	$5.1^{\diamond}$	[Strohm et al., 2005] [Inyushkin and Taldenkov, 2007]
	$(Tb_{.3}Y_{.7})_3Ga_5O_{12}$	0.1	$9.5 \cdot 10^{-2}$	10	$8.0^{\star}$	[Hirokane et al., 2019]
Spin liquid candidates	dmit-131	0.05	?	0–10	$2^{\star}$	[Bourgeois-Hope et al., 2019] [Ni et al., 2019]
		See also [Yamashita et al., 2010] where quantitatively different values are obtained.				
	α-$RuCl_3$	15	8	12	$20^{\star}$	[Kasahara et al., 2018b]
		8	3.5	16	35	[Hentrich et al., 2019]
		See also [Czajka et al., 2022, Bruin et al., 2022a] for consistent results at lower T.				
Cuprates	La_2CuO_4	12	−38	15	$20^{\star}$	[Grissonnanche et al., 2019]
	$Sr_2CuO_2Cl_2$	7	−21	15	$20^{\star}$	[Boulanger et al., 2020]
	Nd_2CuO_4	56	−200	15	$20^{\star}$	[Boulanger et al., 2020]
	Note that doped cuprates are explored in [Grissonnanche et al., 2020, Boulanger et al., 2022].					
Kagome materials	Cu(1,3-bdc)	≈ 0.07	≈ 0.2	0.1	$0.82^{\star}$	[Hirschberger et al., 2015a]
	Volborthite	1.9	−0.66	15	$22^{\star}$	[Watanabe et al., 2016]
	Ca-Kapellasite	≈ 0.2	≈ 1	15	$20^{\star}$	[Doki et al., 2018]
	Cd-Kapellasite	1.7	11	15	$10^{\star}$	[Akazawa et al., 2020]

Evolution of the experimentally observed THE

Note that although the κ_{xy} in these materials differs in three orders of amplitude, while the ratio of κ_{xy} and κ_{xx} which is called thermal Hall angle are not so different around few per thousand



Thermal conductivity of phonons

Similarly to the electronic one:

$$\kappa_{ph} = \frac{1}{3}\int_0^{\frac{\theta_D}{T}} C(x)vl(x)dx$$

Ballistic: mean-free-path l of the phonons becomes comparable with the sample size
T^3 thermal conductivity

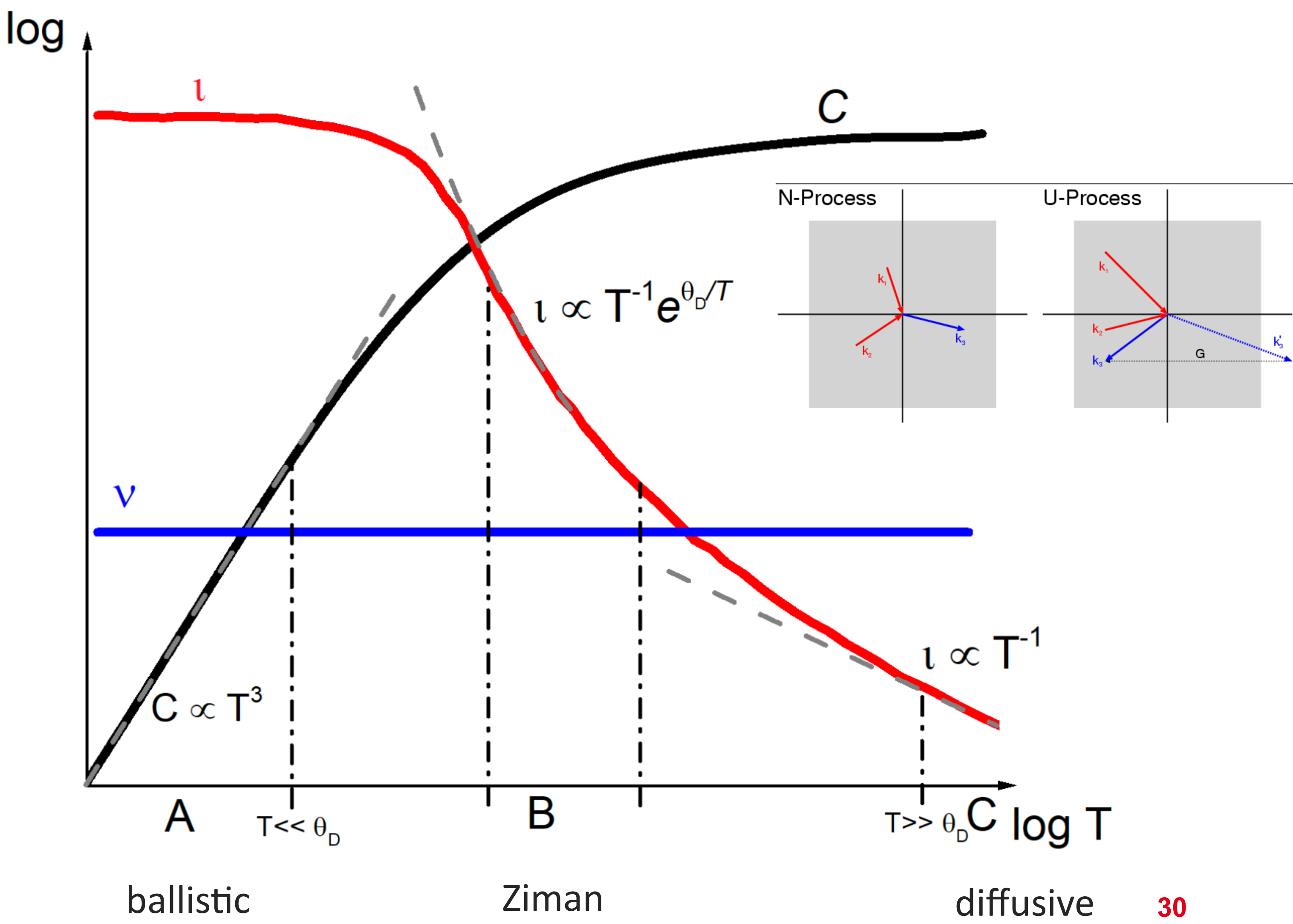
Ziman: Few Umklapp process in phonon-phonon scattering $\tau \propto T^{-1}e^{\theta_D/T}$

$\kappa \propto e^{\theta_D/aT}$.

Diffusive: Dominance of Umklapp scattering.

$\tau \propto T^{-1}$

T^{-1} thermal conductivity

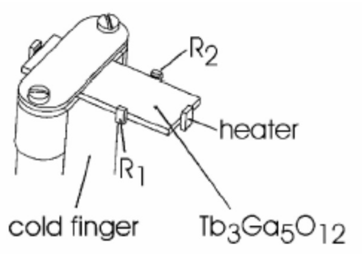


THERMAL HALL EFFECT OF PHONONS :

Extrinsic THE: $Tb_3Ga_5O_{12}$

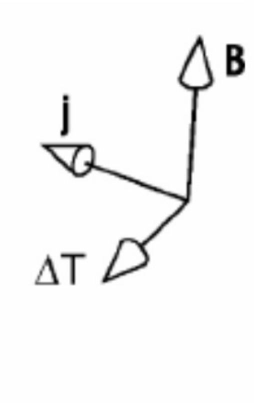
First discovered in paramagnetic insulator $Tb_3Ga_5O_{12}$: skew scattering of phonons

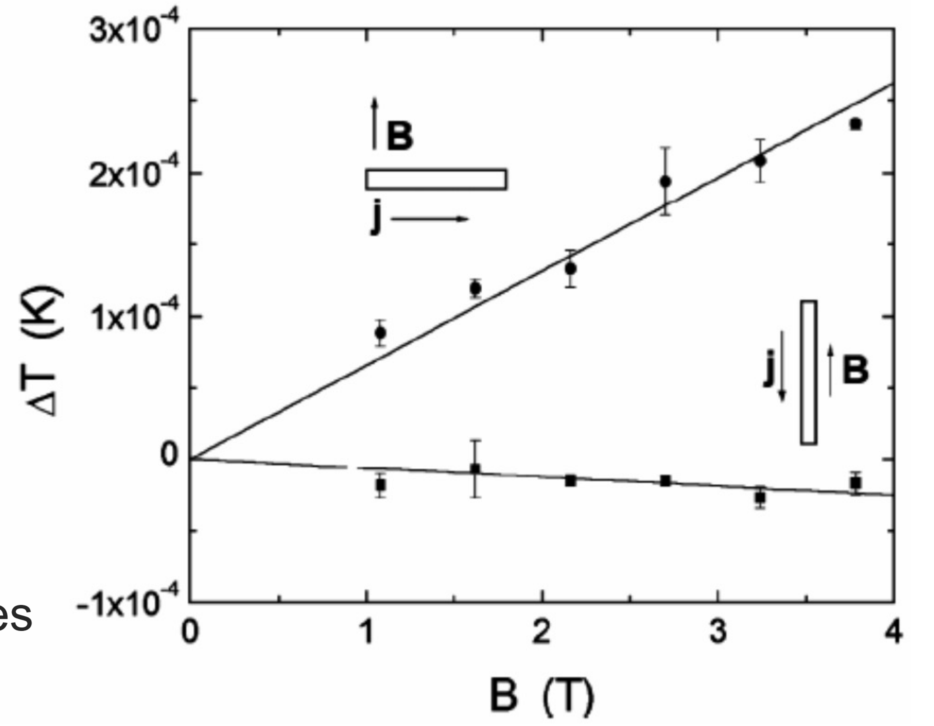
Spin-phonon interaction



$$\kappa_H \sim C_v \, v^2 \, \tau^2 \tau_o^{-1}$$

time-reversal odd *skew scattering* rate





Skew scattering of phonons by superstoichiometric Tb magnetic impurities

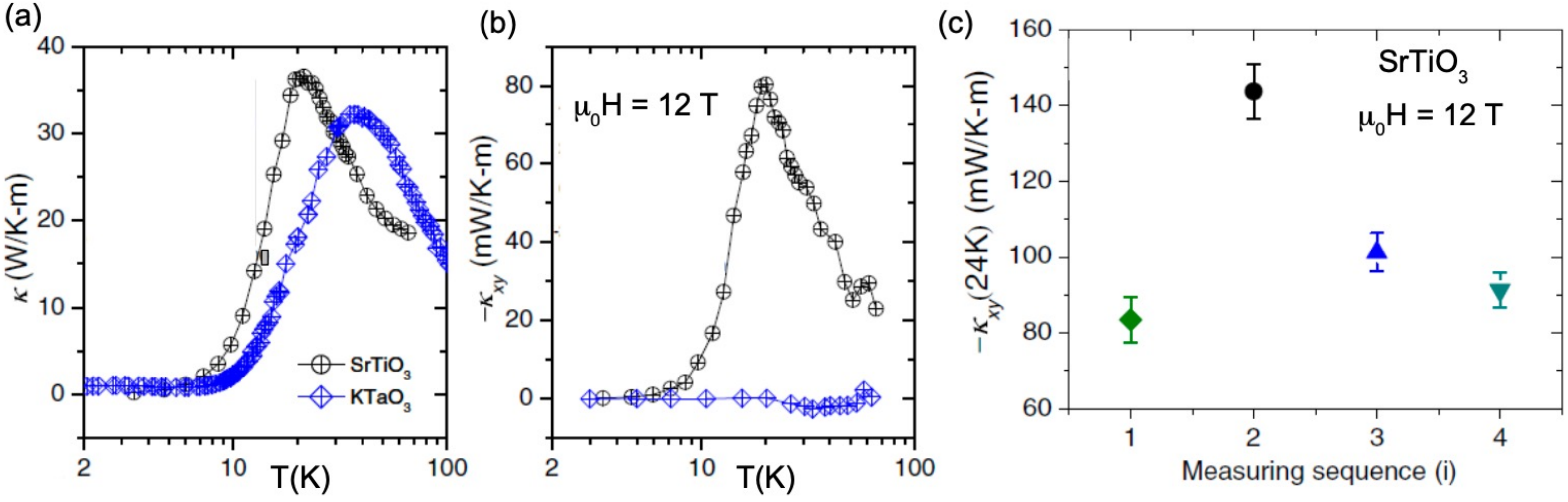
C. Strohm *et al.*, Phys. Rev. Lett. **95**, 155901(2006).

A.V. Inyushkin *et al.*, JETP Lett. **86**, 379 (2007).

THERMAL HALL EFFECT OF PHONONS

$SrTiO_3$: extrinsic κ_{xy} related to skew scattering or intrinsic κ_{xy} linked to ferroelectric fluctuations (Berry phase)?

The intrinsic κ_{xy} is still orders of magnitude smaller than the observed effect, and the extrinsic κ_{xy} proportional to the phonon mean-free path appears likely to explain the observations



Importance of the
Antiferro distortive
transition ?

cubic to tetragonal unit cell

LI X, FAUQUÉ B, ZHU Z, BEHNIA K, . Phys. Rev. Lett., **124,** 105901 (2020).

How to get the origin of the THE

1/ Mesure all the thermal, thermo-electric and electric coefficients

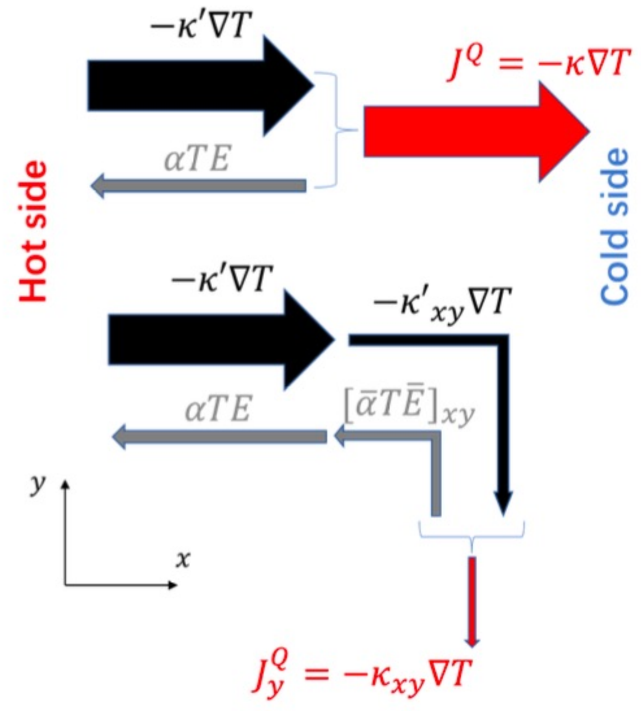
$$\vec{J}^e = \overline{\sigma'}\vec{E} - \overline{\alpha}\vec{\nabla}T$$

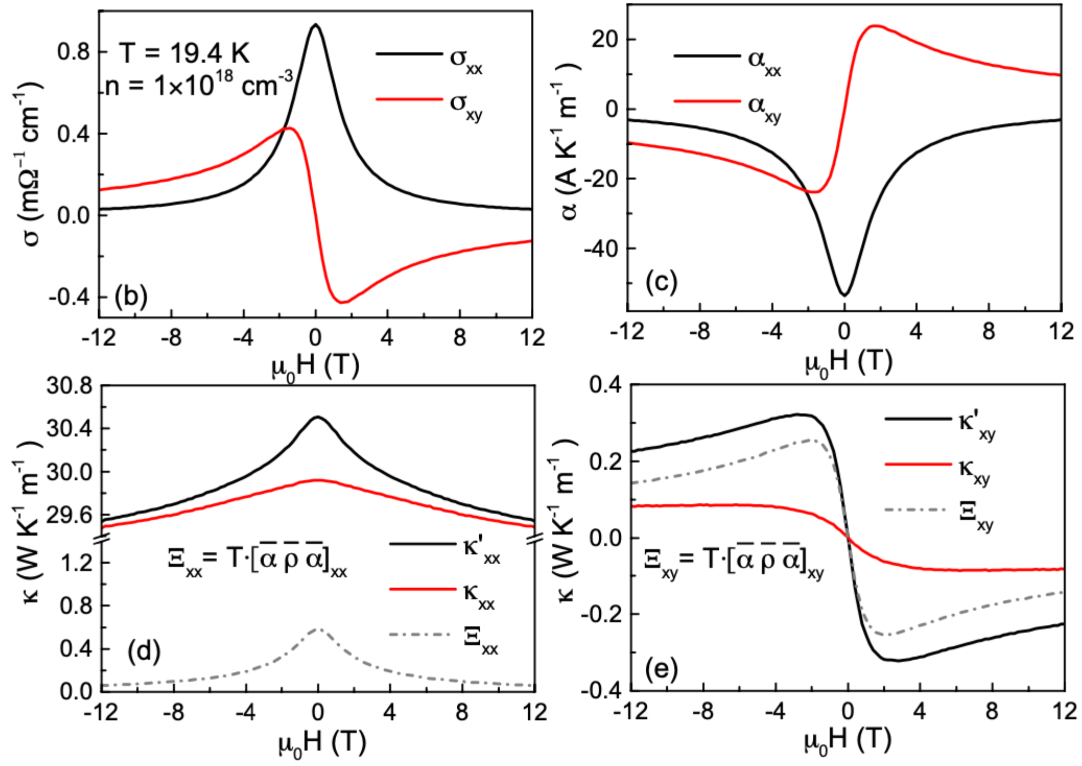
$$\vec{J}^Q = \overline{\alpha}T\vec{E} - \overline{\kappa'}\vec{\nabla}T$$

In the case of a thermal experiment where

$$\vec{J}^Q \neq \vec{0},\ \vec{J}^e = \vec{0}$$

$$\vec{J}^Q = \underbrace{(\overline{\alpha}\,\overline{\rho}\,\overline{\alpha}\,T - \overline{\kappa'})}_{\kappa_{xy}}\vec{\nabla}T$$





κ_{xy} and κ'_{xy} are significantly different

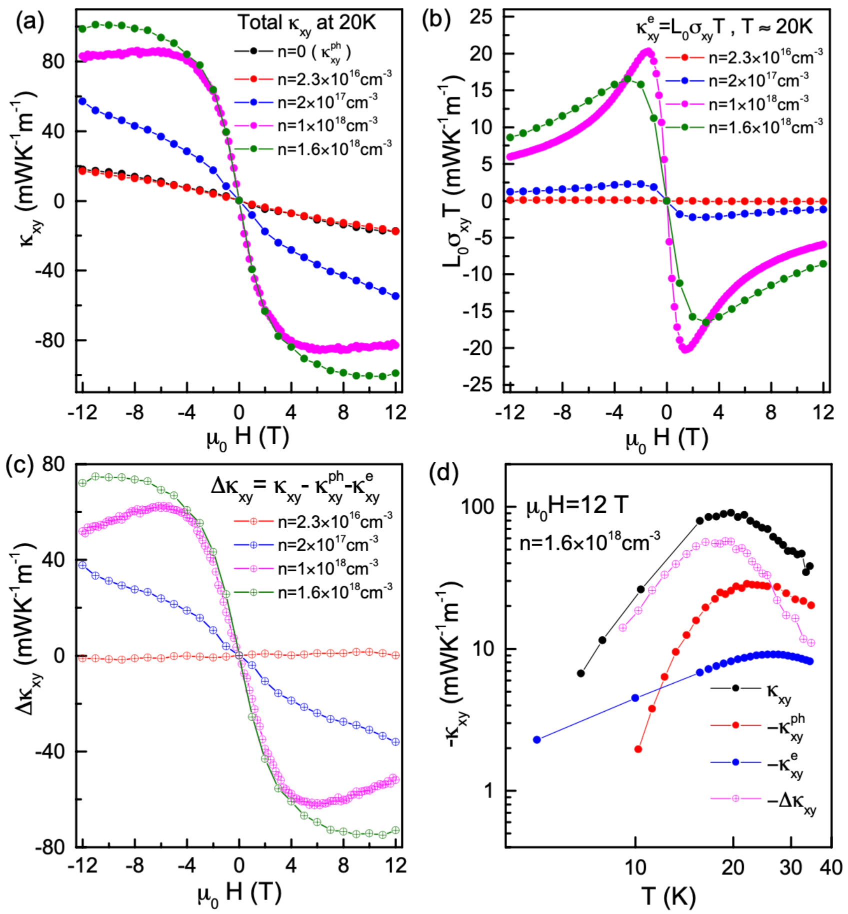
Consequence of a large Hall angle ($\sigma_{xx} \approx \sigma_{xy}$), large thermoelectric power and Nernst effect

How to get the origin of the THE

2/ Wiedemann-Franz law to estimate the electronic contribution

$$\kappa_{xy}^{e} = L_0 \sigma_{xy} T$$

$$\Delta\kappa_{xy} = \kappa_{xy} - \kappa_{xy}^{ph} - \kappa_{xy}^{e}$$



LI X, FAUQUÉ B, ZHU Z, BEHNIA K . Phys. Rev. Lett., **124,** 105901 (2020).

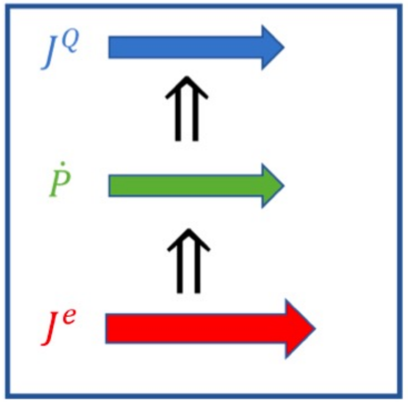
Thermal Hall Effect of Phonons

Drag on phonons by the electric current

$$\kappa_{xy}(drag) = -\alpha_{xy}\frac{m^* v_s^2}{e} f \frac{\tau_p}{\tau_e}$$

The component of thermal Hall conductivity generated by mutual drag between electrons and phonons is proportional to the product of α_{xy}, the ratio of phonon and electron τ_e scattering times τ_p and the efficiency of momentum transfer between the two baths

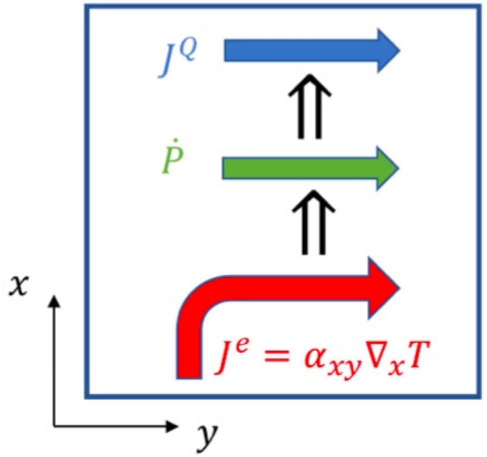
Peltier phonon drag



Electron flow J^e triggers momentum flow $\dot{P}$, which triggers a phonon energy flow J^Q :

$$\Pi_{drag} = \frac{J^Q}{J^e}$$

Thermal Hall phonon drag



J^e ,generated by a thermal gradiant perpendicular to it, will similarly lead to J^Q

$$\kappa_{xy}^{drag} = -\alpha_{xy}\Pi_{drag}$$

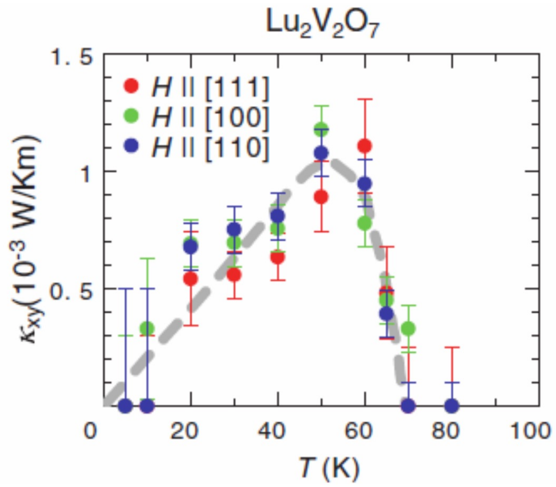
Herring Phys. Rev. 96, 1163 (1954)

Magnon Thermal Hall Effect

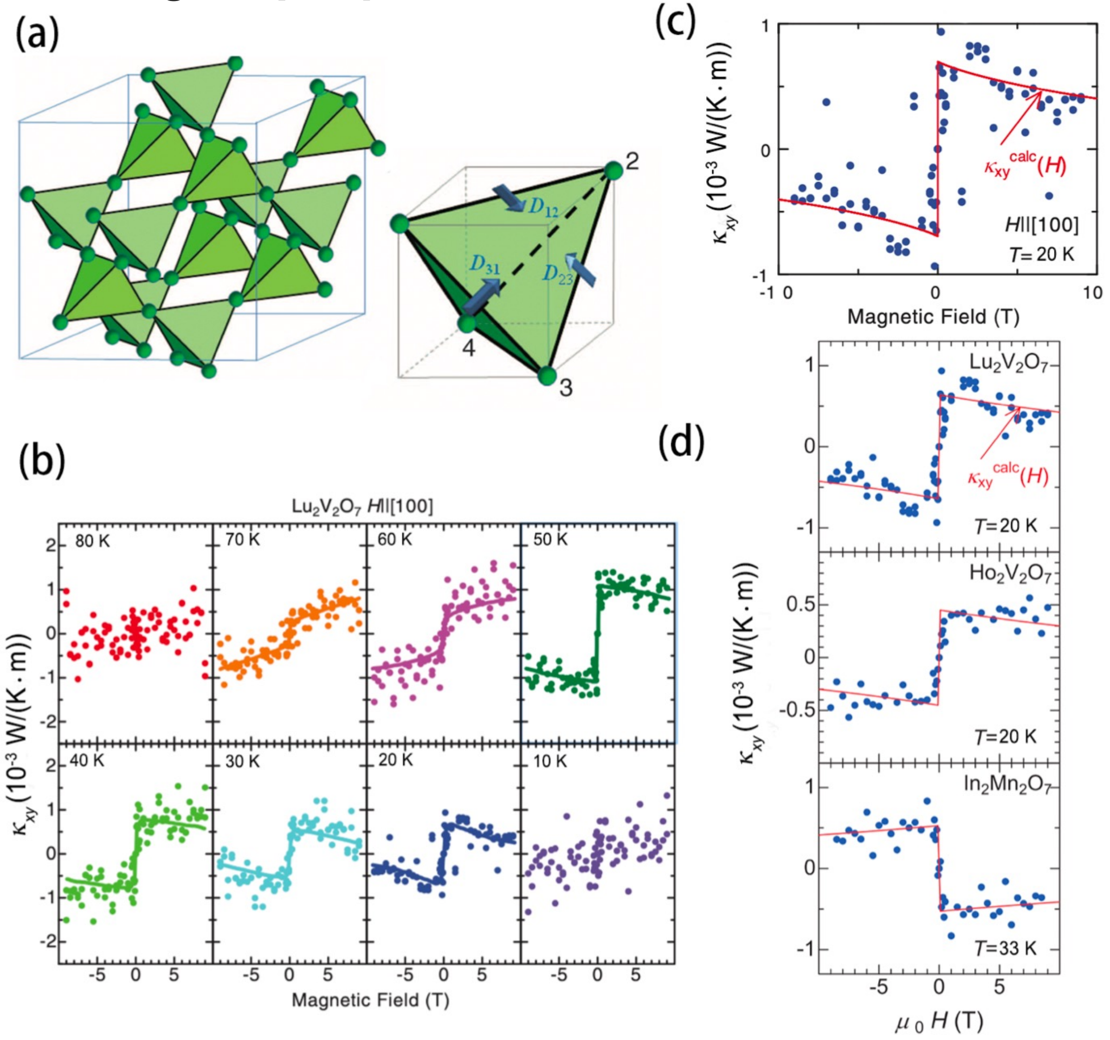
$Lu_2V_2O_7$: Insulating collinear ferromagnet T_c=70 K

The heat carriers can only be phonons or magnons

$$H_{\mathrm{DM}} = \sum_{i,j} \boldsymbol{D}_{ij} \cdot (\boldsymbol{S}_i \times \boldsymbol{S}_j).$$



Stacking of alternating kagome and triangular lattices along the [111]



Population of magnons decreases as the magnetic field increases

Y. Onose, *et al.*, Science **329**, 297 (2010).

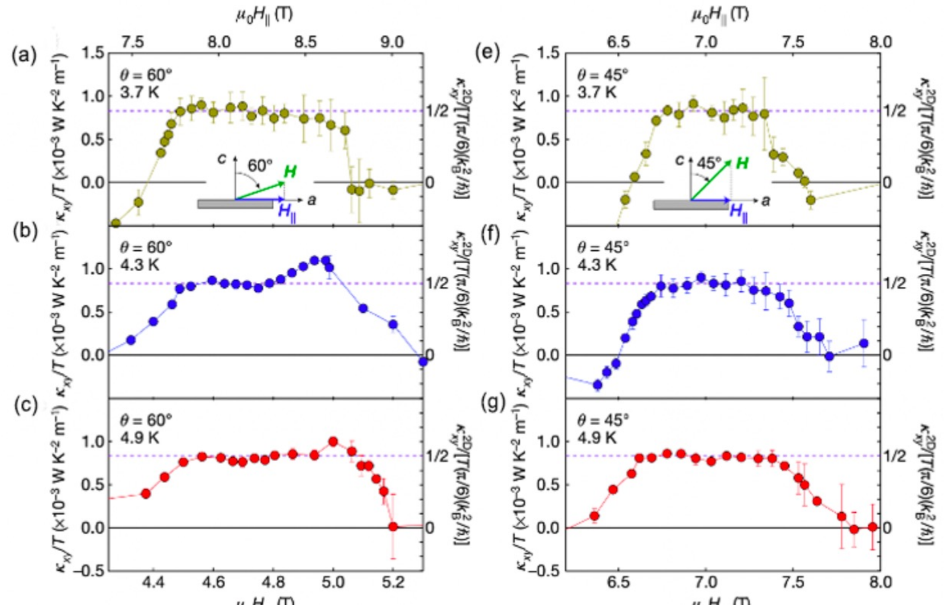
Materials with controversial origins of THE

$RuCl_3$ Quantum spin liquid

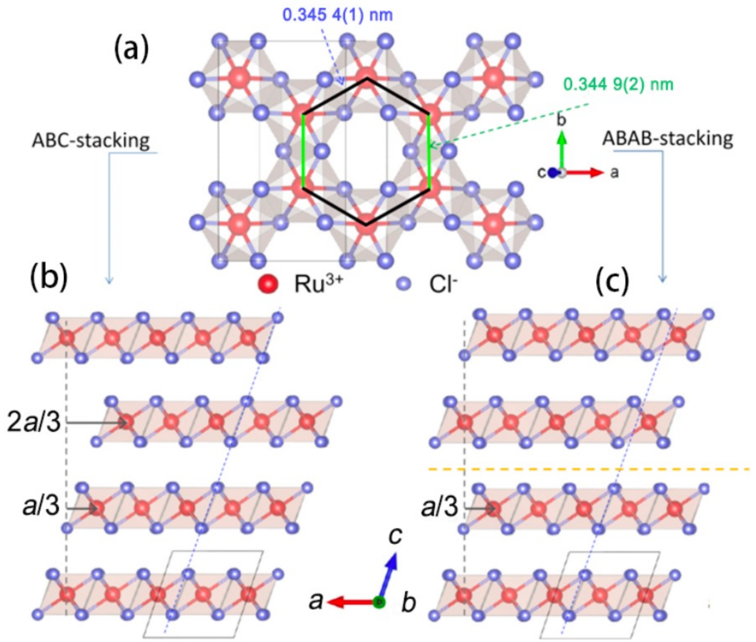
3 types of Heat carriers:

1/ Majorana fermions predicted to occur in Kitaev quantum spin liquid

$$\kappa_{xy}^{2D}/T = c(\pi/6)\left(k_B^2/\hbar\right)$$



YOKOI et al. Science 373 568 (2021)



Observation of a plateau in the THE???

2/ Phononic skew scattering THE

HENTRICH et al. Phys. Rev. Lett., 120: 117204 (2018)

3/ Electronic THE: quantum oscillation in k_{xx}

CZAJKA P et al. Nat. Phys., 17, 915 (2021)

Materials with controversial origins of THE

Cuprate High-Temperature Superconductors

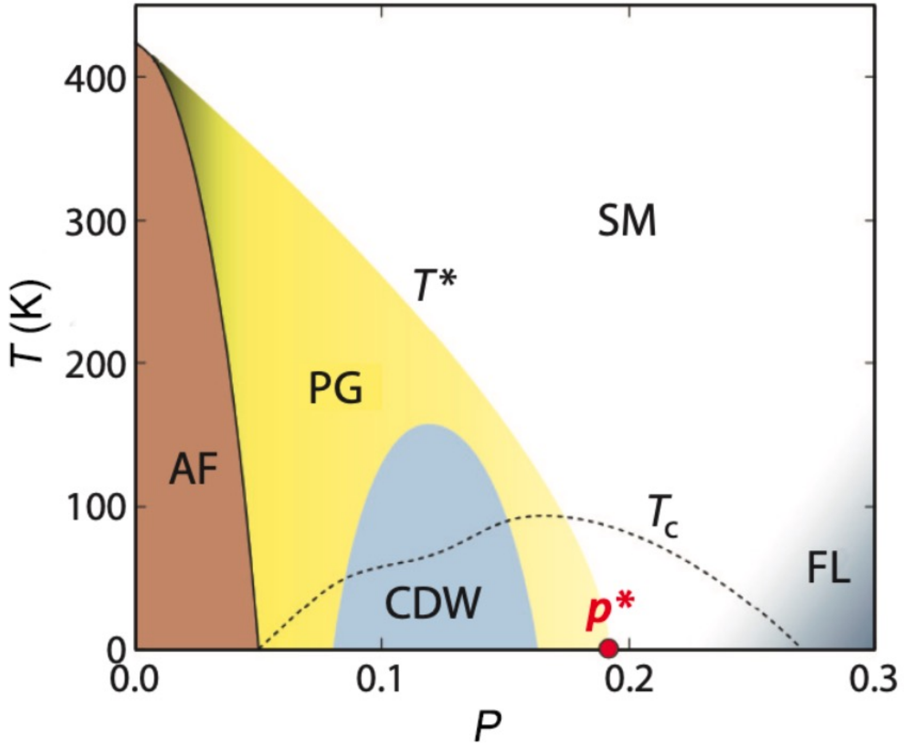
Chiral phonons ?

Without doping : Mott insulators

Overdoped regime : Fermi liquid

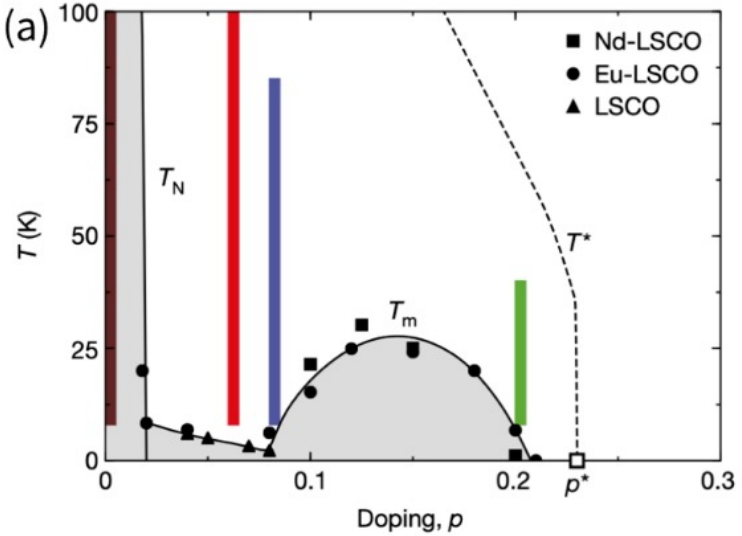
Intermediate regime : the pseudogap phase many unusual

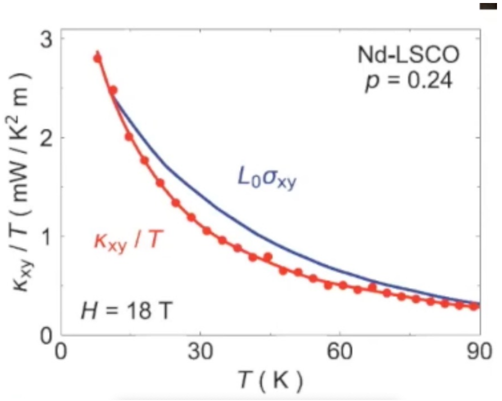
phenomena



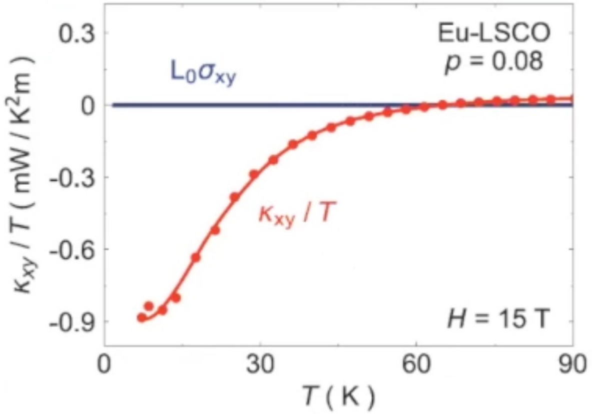
Materials with Controversial Origins of THE

Cuprate High-Temperature Superconductors



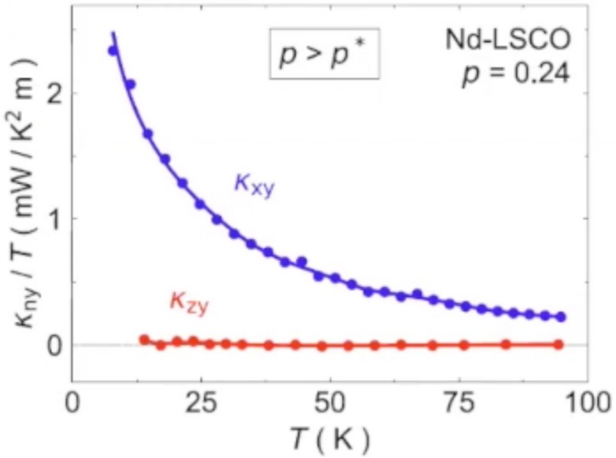


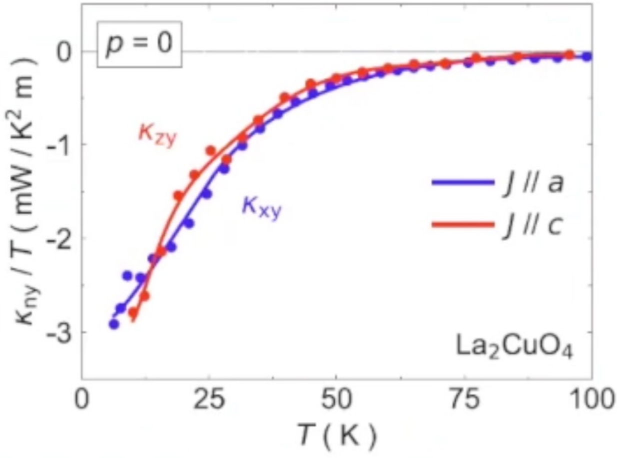
Electron THE



Phonon THE

No electronic THE perpendicular to CuO plane





Grisonnanche et al. Nature 571, 376 (2019)

Phonon THE

Materials with Controversial Origins of THE

Cuprate High-Temperature Superconductors

1/ electrons ? since the parent compound, a Mott insulator generate large signals, NO.

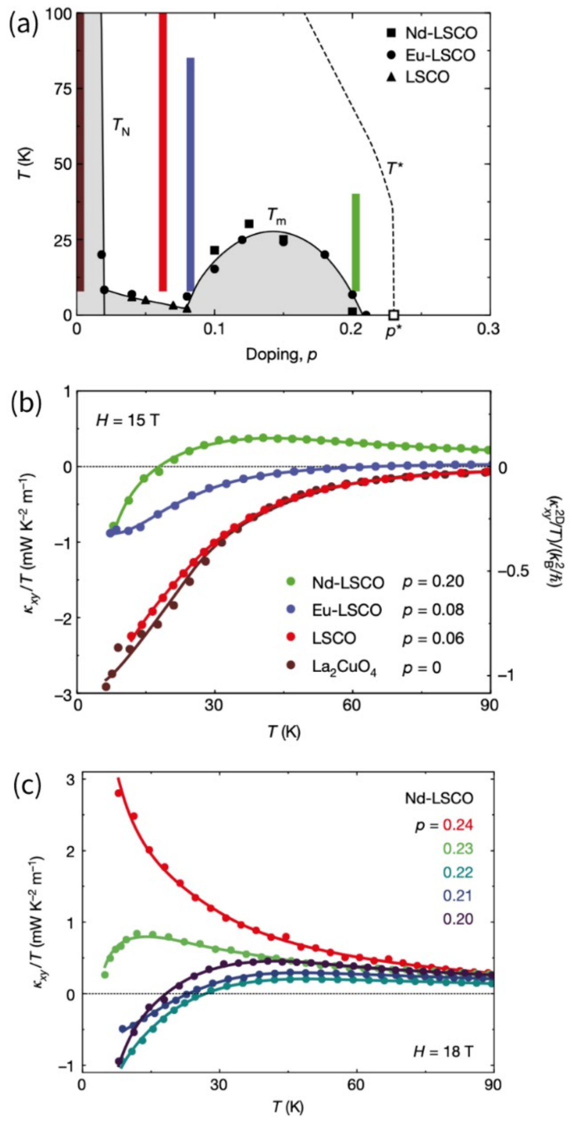
2/ magnons ? as κ_{xy} remains even without static magnetism, NO.

3/ **phonons :**Isotropy of the thermal Hall conductivity : **YES**

Phonons become chiral in the pseudo gap phase

Many-body scattering of phonons by collective modes:
a consequence of non- Gaussian correlation (skew scattering)

Mangeolle, Phys. Rev. X **12**, 041031 (2022)



Grisonnanche et al. Nature 571, 376 (2019)

New perspective THE in p-wave superconductors

A superconductor is chiral (spin-triplet) when it breaks time-reversal symmetry (TRS) due to its orbital degrees of freedom

First signature : A small internal magnetic field below the critical superconducting temperature, arising from the finite orbital momentum, detected by muon spectroscopy, Kerr effect, knight shift in NMR measurements

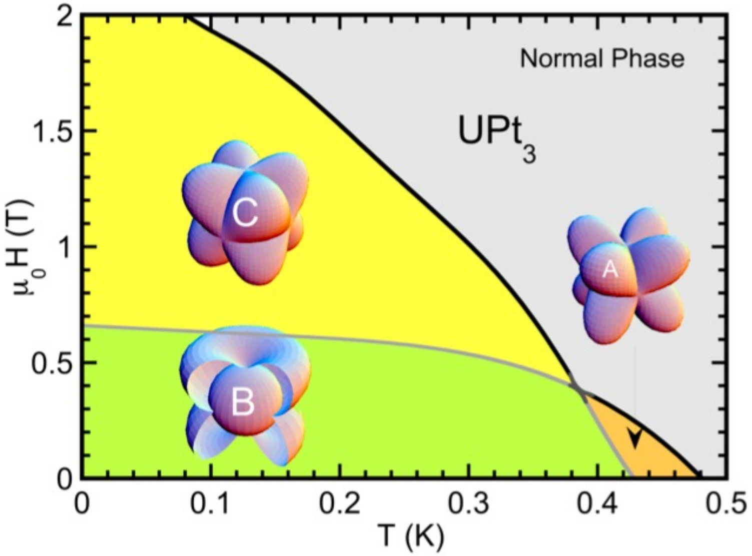
Alternative signature :

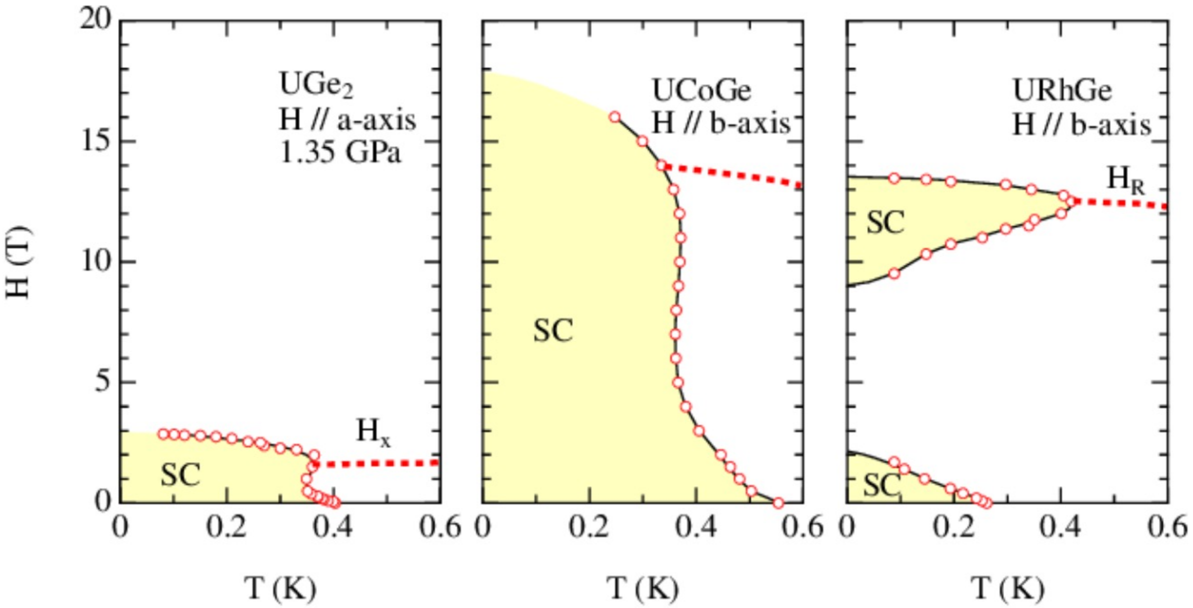
Anomalous bulk thermal Hall effect is predicted in chiral *p*-wave superconductors

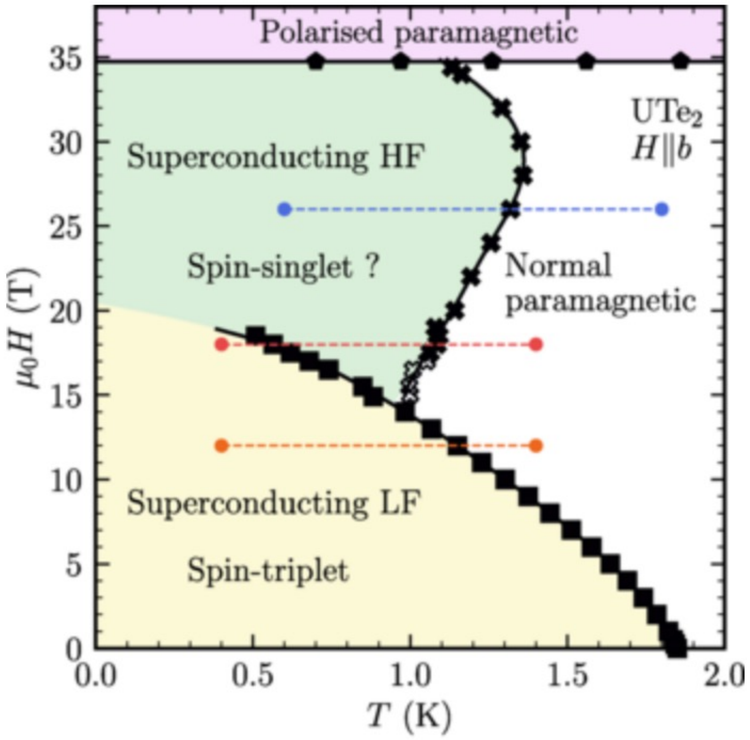
Quantized thermal Hall at zero field from the presence of edge heat currents similar to the edge charge currents of the integer quantum Hall effect

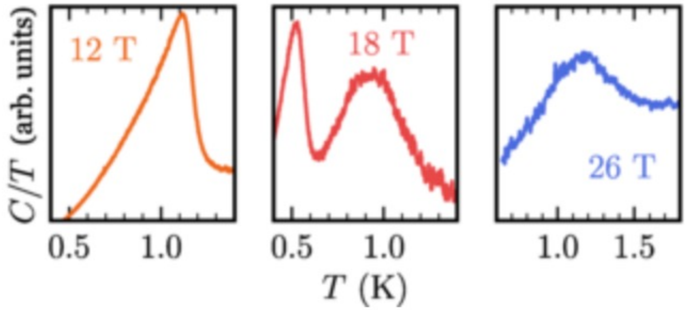
$$\kappa_{x,y} = \frac{\pi k_B^2 T}{12\hbar} = \frac{1}{4}\mathcal{L}_0 G_0 T$$

New perspective THE in p-wave superconductor









Summary

- The thermal Hall effect, plays and will continue to play a major role in characterizing insulating quantum materials.
- Both phonons and pure spin excitations can contribute to k_{xy}, where the underlying mechanisms vary, but a common feature is that heat carriers are subject to chirality in one way or another.
- Phonons can be conferred chirality through various extrinsic and intrinsic mechanisms. The extrinsic mechanisms are related to the scattering by rare-earth impurities and by structural domain boundaries, while the intrinsic mechanisms include the Berry curvature and coupling of phonons to spins.
- Besides phonons, spin excitations such as magnons also play a critical role in the thermal Hall effect in insulating magnets. Dzyaloshinskii–Moriya interaction is a dominant mechanism of the magnon thermal Hall effect.
- Anomalous bulk thermal Hall effect is predicted in chiral *p*-wave superconductors

Thank you !

Charge and Heat Transport coefficients

With electrical field:

$$\frac{df_k(\vec{r})}{dt}|_{\vec{E}} = \frac{\partial f_k(\vec{r})}{\partial \epsilon_k}\frac{\partial \epsilon_k}{\partial \vec{k}}\frac{\partial \vec{k}}{\partial t} = \frac{\partial f_k(\vec{r})}{\partial \epsilon_k}\hbar \vec{v}_k \frac{e\vec{E}}{\hbar} \approx \frac{\partial f^0}{\partial \epsilon_k}\vec{v}_k e\vec{E}$$

Group velocity $\vec{v}_k = \frac{1}{\hbar}\frac{\partial \epsilon_k}{\partial \vec{k}}$

With a thermal gradient:

$$\frac{df_k(\vec{r})}{dt}|_{\vec{\nabla}T} = \frac{\partial f_k(\vec{r})}{\partial T}\frac{\partial T}{\partial \vec{r}}\frac{\partial \vec{r}}{\partial t} = \frac{\partial f_k(\vec{r})}{\partial T}\vec{v}_k \vec{\nabla} T \approx \frac{\partial f^0}{\partial T}\vec{v}_k \vec{\nabla} T$$

With thermal gradient and electrical field:

$$f_k(\vec{r}) = f^0 - \tau \vec{v}_k\left(\frac{\partial f^0}{\partial \epsilon_k}e\vec{E} + \frac{\partial f^0}{\partial \epsilon_k}\frac{\epsilon_k - \mu}{T}\vec{\nabla}T\right)$$

$$\frac{\partial f^0}{\partial T} = -\frac{\partial f^0}{\partial \epsilon_k}\frac{\epsilon_k - \mu}{T}$$

Perturbated distribution function under electrical field and thermal gradient

Anomalous Hall effect

Kubo formula:

$$\sigma_{xy}(\omega) = \mathrm{i} \sum_{k,n,m} \frac{\langle mk|J_x|nk\rangle \langle nk|J_y|mk\rangle}{\varepsilon_n(k) - \varepsilon_m(k)} \times \frac{f(\varepsilon_n(k)) - f(\varepsilon_m(k))}{\omega + \mathrm{i}\delta + \varepsilon_m(k) - \varepsilon_n(k)},$$

$$\langle nk|J_\mu|mk\rangle = (\varepsilon_m(k) - \varepsilon_n(k))\langle nk|\partial/\partial k_\mu|mk\rangle$$

Overlap integral of the two Bloch wave functions at neghboring points in k-space

Charge and Heat Transport coefficients

Transverse transport in magnetic field

$$\hbar\frac{df_k(\vec{r})}{dt}\Big|_{\vec{B}} = \hbar\frac{df_k(\vec{r})}{d\vec{k}}\frac{d\vec{k}}{dt} = e(\vec{v_k}\times\vec{B})\frac{df_k(\vec{r})}{d\vec{k}}$$

The distribution function in the presence of a magnetic field

$$f_k(\vec{r}) = f_k^0(\vec{r}) + g_k(\vec{r}) \qquad e(\vec{v_k}\times\vec{B})\frac{df_k^0(\vec{r})}{d\vec{k}} = e(\vec{v_k}\times\vec{B})\cdot\hbar\vec{v_k}\frac{df_k^0(\vec{r})}{d\epsilon_k} = 0$$

(Only true if Landau quantization is negligible)

With thermal gradient, electrical field and magnetic field:

$$\frac{df_k(\vec{r})}{dt} = \frac{e}{\hbar}(\vec{v_k}\times\vec{B})\frac{\partial g_k(\vec{r})}{\partial\vec{k}} + \frac{\partial f^0}{\partial\epsilon_k}\vec{v}_k e\vec{E} + \frac{\partial f^0}{\partial T}\vec{v}_k\vec{\nabla}T = -\frac{g_k(\vec{r})}{\tau}$$

$$g_k(\vec{r}) = -\mu(\vec{v_k}\times\vec{B})\frac{\partial g_k(\vec{r})}{\partial\vec{v}} - e\tau\frac{\partial f^0}{\partial\epsilon_k}\vec{v}_k\vec{E} - \tau\frac{\partial f^0}{\partial T}\vec{v}_k\vec{\nabla}T$$

$$= -\vec{v_k}\cdot\left(\mu\vec{B}\times\frac{\partial g_k(\vec{r})}{\partial\vec{v}} + e\tau\frac{\partial f^0}{\partial\epsilon_k}\vec{E} + \tau\frac{\partial f^0}{\partial T}\vec{\nabla}T\right)$$

With the mobility:

$$\mu = \frac{e\tau}{m} = \frac{e\tau}{\hbar}\frac{\partial v}{\partial k}$$