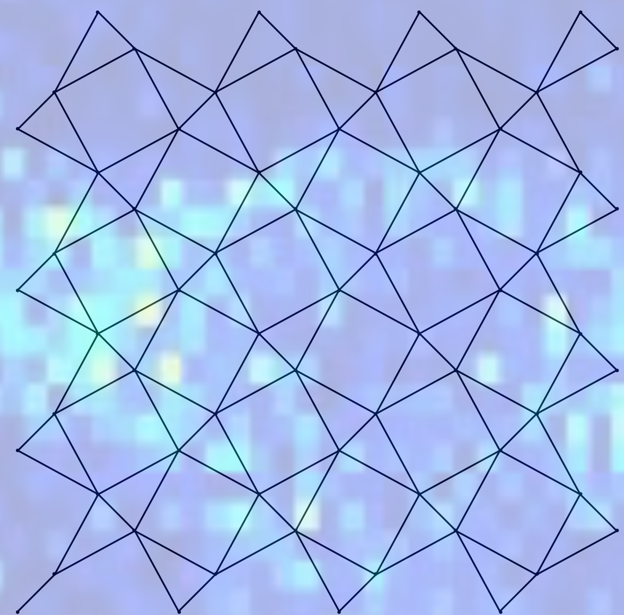


A Tutorial on Topological Magnons

Paul McClarty
Laboratoire Léon Brillouin, CEA and CNRS, Saclay

GDR School on Topology, Aussois, May 29th 2024

Overview

Magnetism and magnons

Topology

Detecting magnon surface states in materials

Interaction effects

Review: PM, Annual Rev. Condensed Matter Physics (2022)

Lodestone

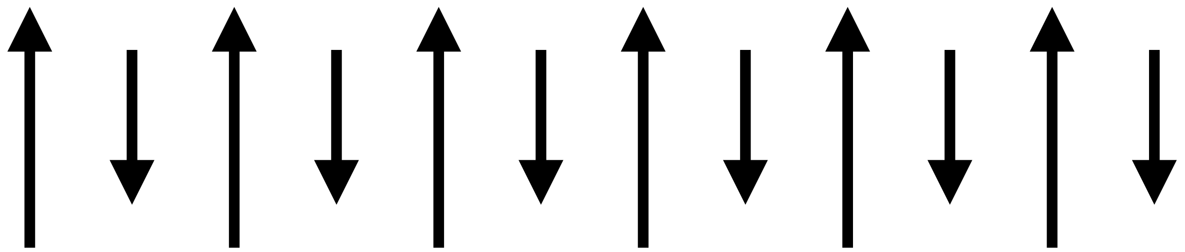
Magnetism

Pierre Curie in the 19th century found that magnetism (magnetic order really) is lost above some critical temperature

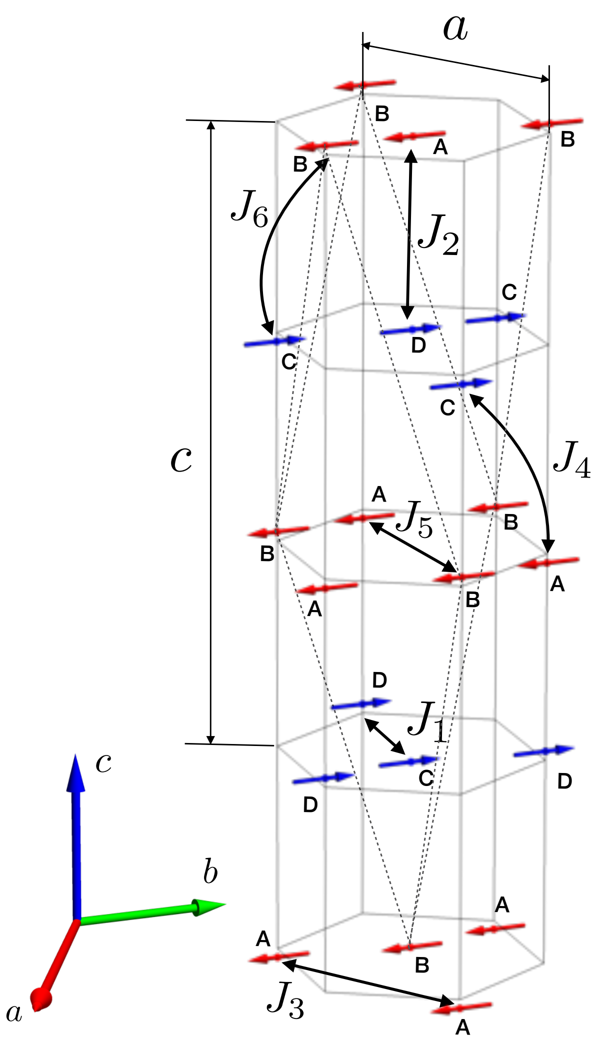
Classical no-go

Phenomenon of magnetic ordering not understood at microscopic level until advent of quantum mechanics

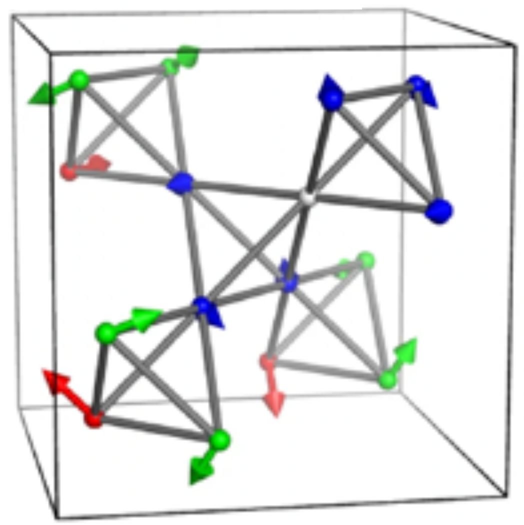
Lodestone is mainly magnetite Fe_3O_4 which is *ferrimagnetic* now famous among condensed matter physicists for frustration and charge order below 120K



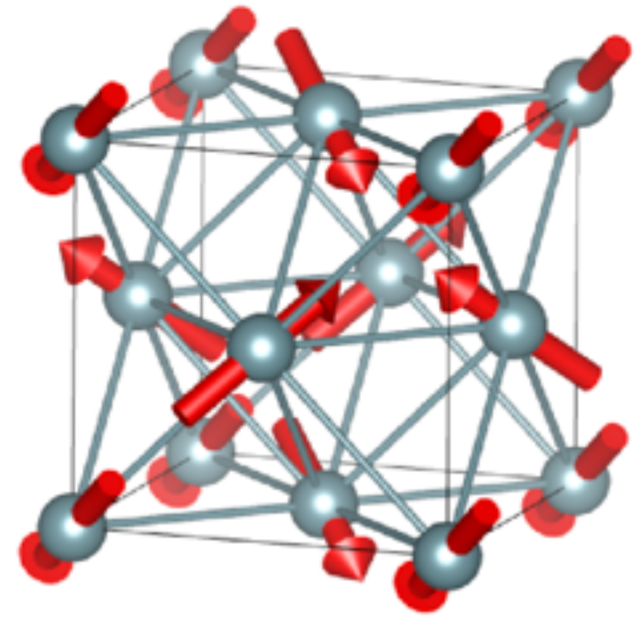
Varieties of Magnetic Ordering



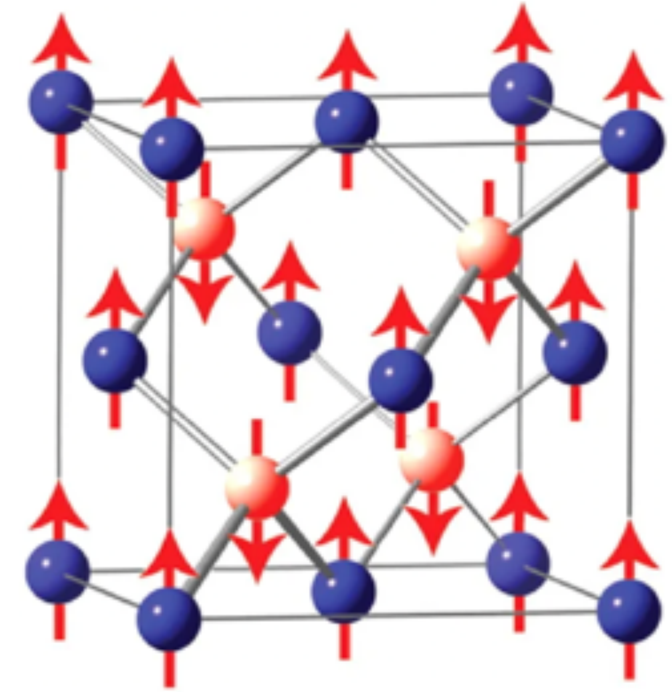
CoTiO3



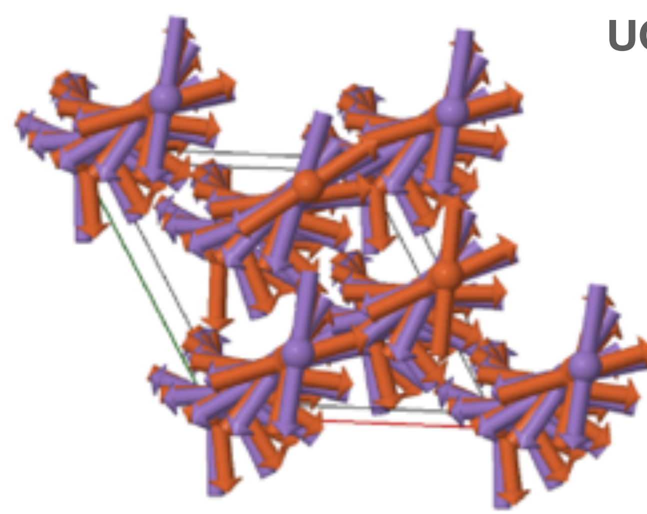
$Gd_2Ti_2O_7$



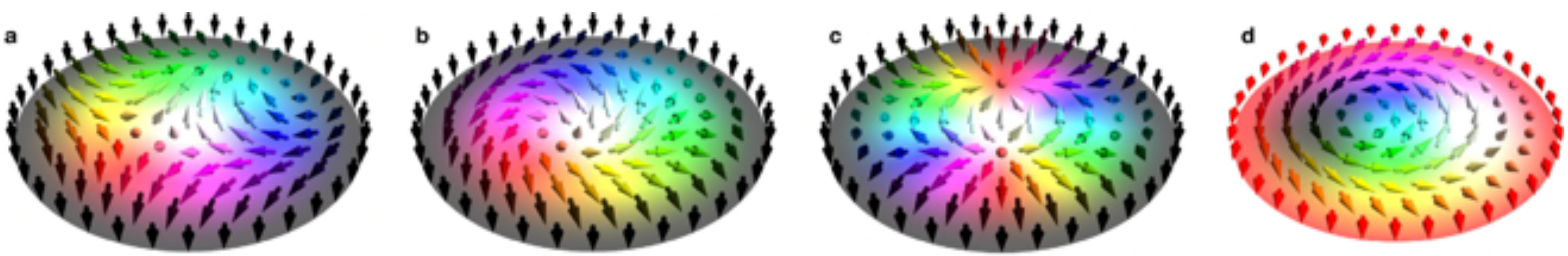
UO_2



Diamond lattice cobalt based antiferromagnet Nagler et al.



Mn_2FeSbO_6



Antiskyrmion

Intermediate skyrmion

Higher-order skyrmion

Bimeron

Mertig et al. review 2018

Spin Waves

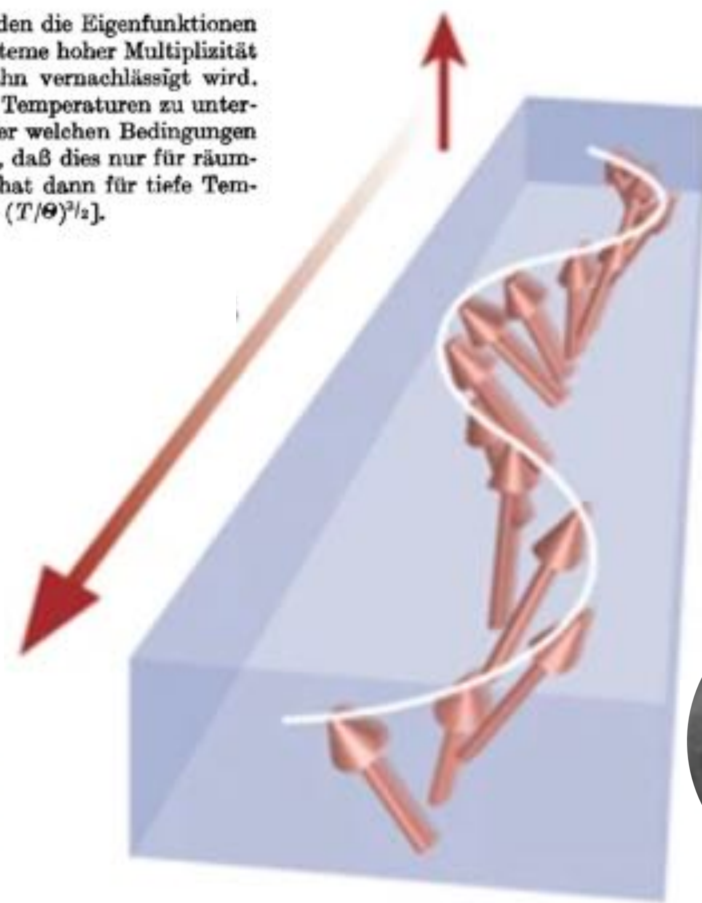
This is a talk about spin waves or **magnons** - collective excitations around some ordered spin texture

Zur Theorie des Ferromagnetismus.

Von **F. Bloch,** zurzeit in Utrecht.

(Eingegangen am 1. Februar 1930.)

Beim Austauschvorgang der Elektronen im Kristall werden die Eigenfunktionen nullter und Eigenwerte erster Näherung für die Termsysteme hoher Multiplizität bestimmt, wobei die Kopplung zwischen Spin und Bahn vernachlässigt wird. Sie gestatten das ferromagnetische Verhalten bei tiefen Temperaturen zu untersuchen und insbesondere die Frage zu beantworten, unter welchen Bedingungen Ferromagnetismus überhaupt möglich ist. Es zeigt sich, daß dies nur für räumliche Gitter der Fall ist; die Sättigungsmagnetisierung hat dann für tiefe Temperaturen die Form $M(T) = M(0)[1 - (T/\Theta)^{3/2}]$.



15, 1940 PHYSICAL REVIEW

Field Dependence of the Intrinsic Domain Magnetization of a Ferromagnet

T. Holstein
New York University, New York, New York
AND
H. Primakoff*
Polytechnic Institute of Brooklyn, Brooklyn, New York

(Received July 31, 1940)

In this paper, the variation of the intrinsic domain magnetization of a ferromagnetic with the external magnetic field, is obtained. The basis of the treatment is the exchange interaction model amplified by explicit consideration of the dipole-dipole interaction between the atomic magnets. Approximations appropriate to low temperatures and equivalent to those used by Bloch in his derivation of the $T^{\frac{3}{2}}$ law, are introduced. The resultant expression for the intrinsic volume susceptibility decreases slowly with increasing field; at high fields the functional dependence is as the inverse square root of the field. The variation with temperature is linear; at room temperature and for fields of about 4000 gauss, the order of magnitude of the (volume) susceptibility is 10^{-4}. The results are compared with experiment and satisfactory agreement is found.

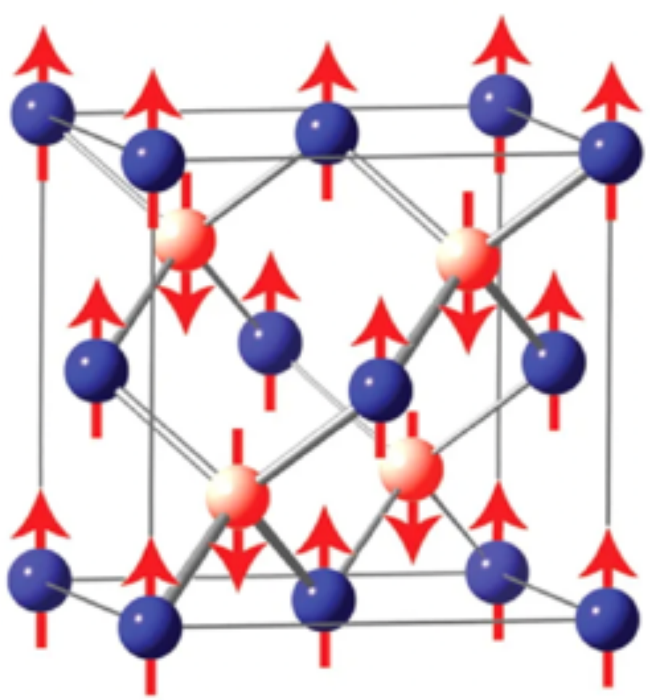
VOLUME 87, NUMBER 4 AUGUST 15, 1952

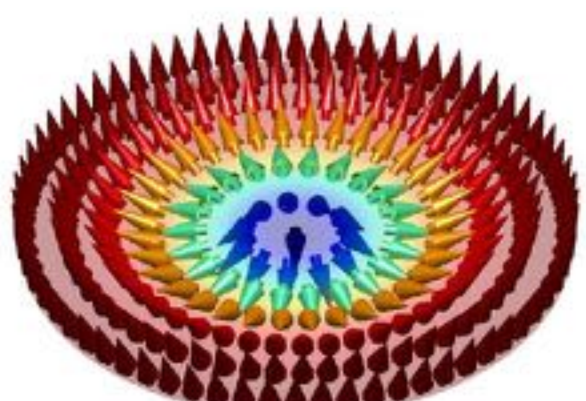
The Spin-Wave Theory of Antiferromagnetics

Ryogo Kubo*
Institute for the Study of Metals, University of Chicago, Chicago, Illinois

(Received March 19, 1952)

The spin-wave theory of antiferromagnets, recently studied by Anderson for the absolute zero of temperature, is examined here for finite temperatures to derive the thermodynamic properties of antiferromagnets at low temperatures. Somewhat differently from Anderson's semiclassical treatment, the present theory has used the formulation devised by Holstein and Primakoff, upon which the thermodynamic quantities are derived quantum-statistically. The parallel susceptibility is shown to be proportional to T^2, while the perpendicular susceptibility is independent of the temperature in the first approximation but decreases with increase in temperature if calculated in the second approximation. A tentative discussion is given of the nature of the divergences which arise in the simple formulation of spin-wave treatments in the absence of any kind of anisotropy.





Using neutrons to probe spin waves

VOLUME 106, NUMBER 5 JUNE 1, 1957

Scattering of Neutrons by Spin Waves in Magnetite

B. N. Brockhouse
General Physics Branch, Atomic Energy of Canada Limited, Chalk River, Ontario, Canada

(Received February 19, 1957)

Measurements of energy distributions of 1.5 A neutrons diffusely scattered by a single crystal of magnetite in the region of the 111 reciprocal lattice point were carried out. Neutron groups were observed which satisfy momentum and energy conservation between the neutron and one wave-excitation quantum, and which are assigned a magnetic origin. The intensities of the neutron groups are consistent with spin wave theory within the limits of the analysis. The measurements are not sufficiently exact to enable the form of the frequency-wave number relation of the spin waves to be deduced, but assuming the quadratic relation of Kaplan a value for the *A-B* exchange integral of 2×10^{-3} ev is obtained.

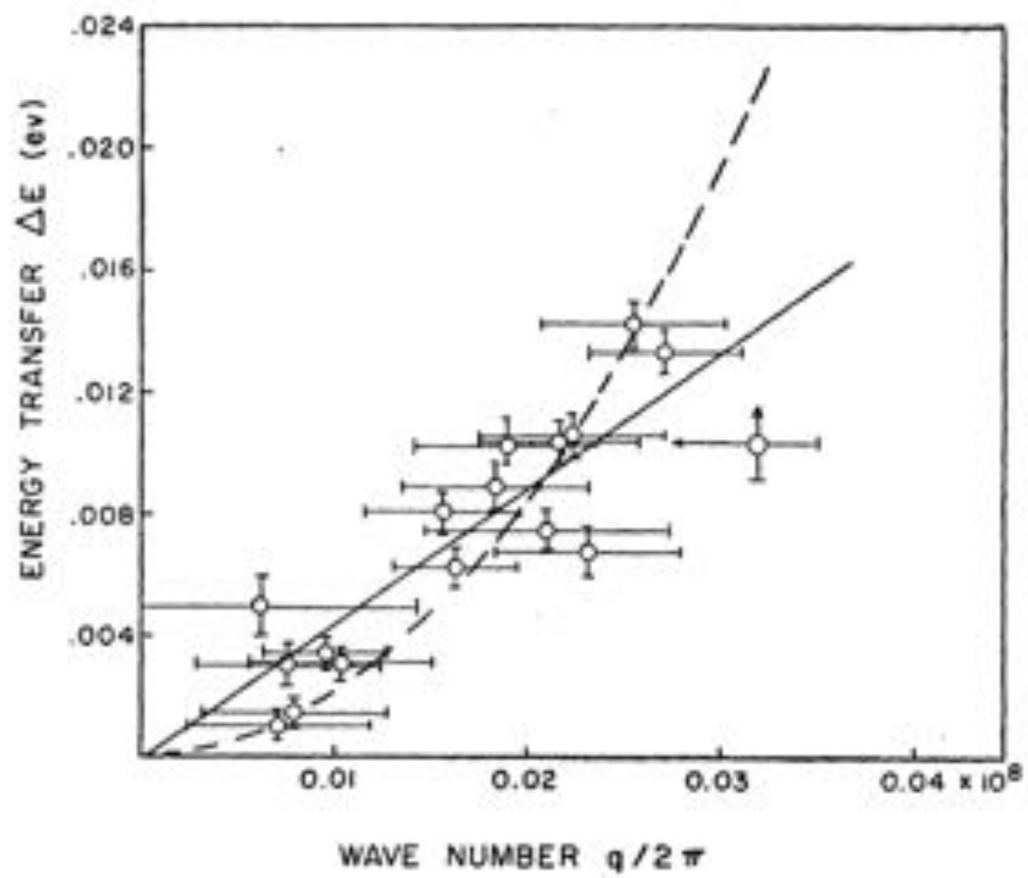
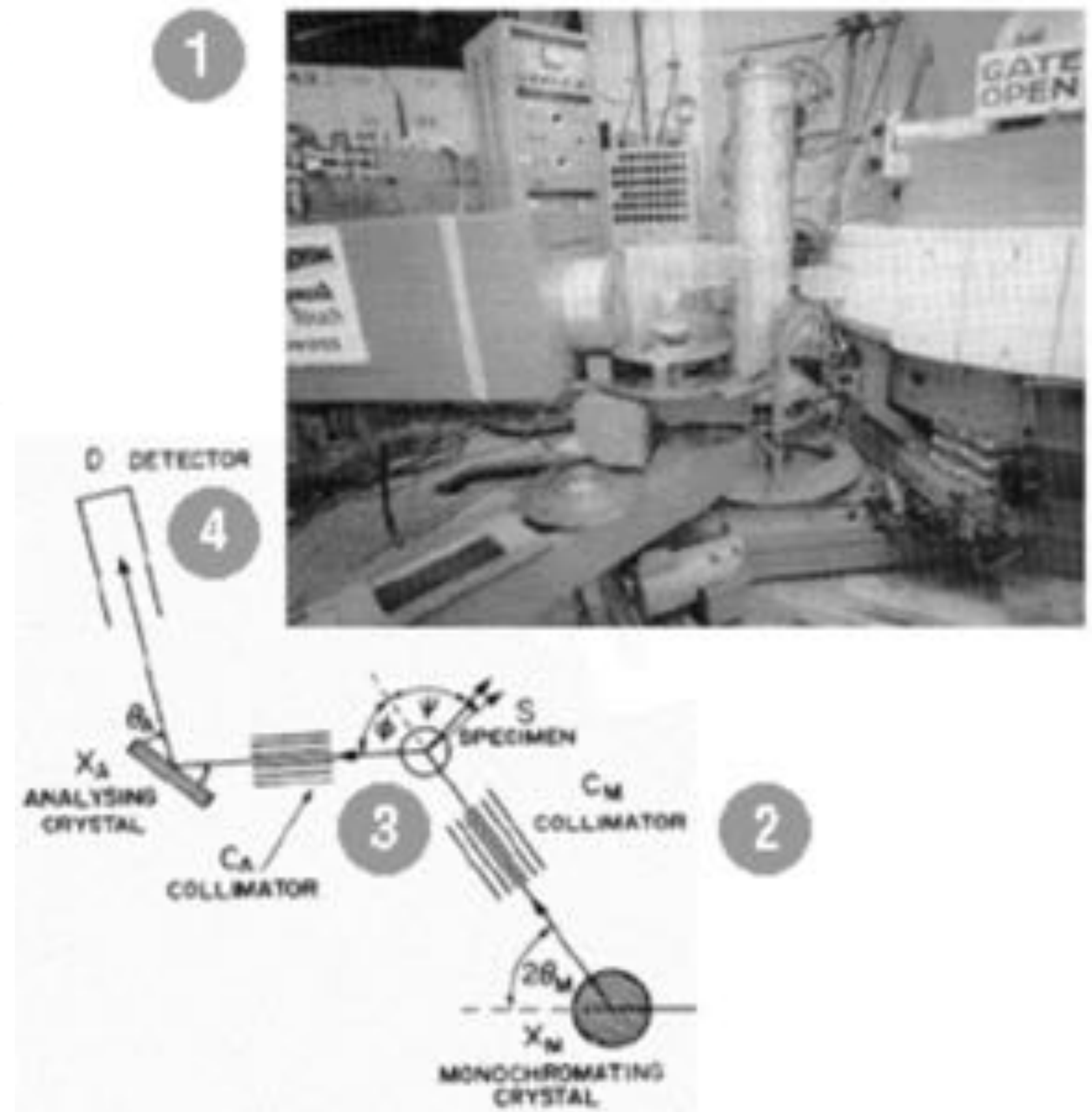
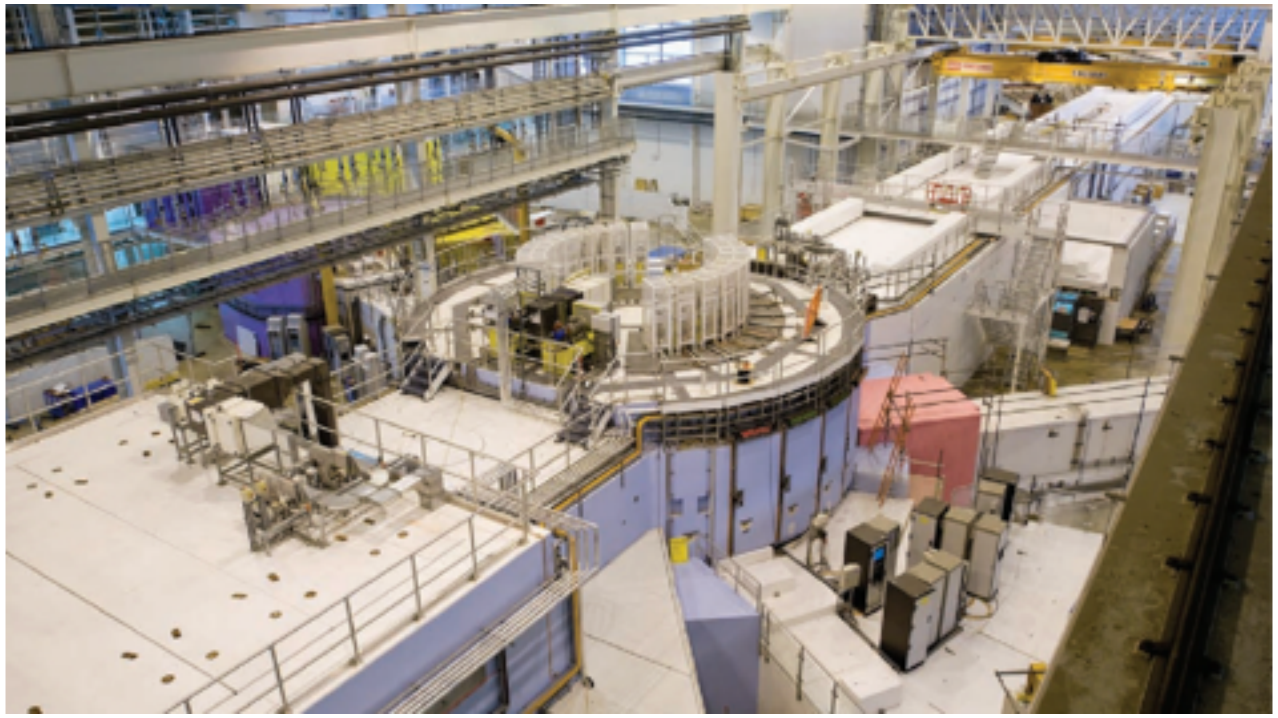


FIG. 5. The energy transfer ΔE as a function of $|\mathbf{q}|/2\pi$, the "spin wave" wave number.

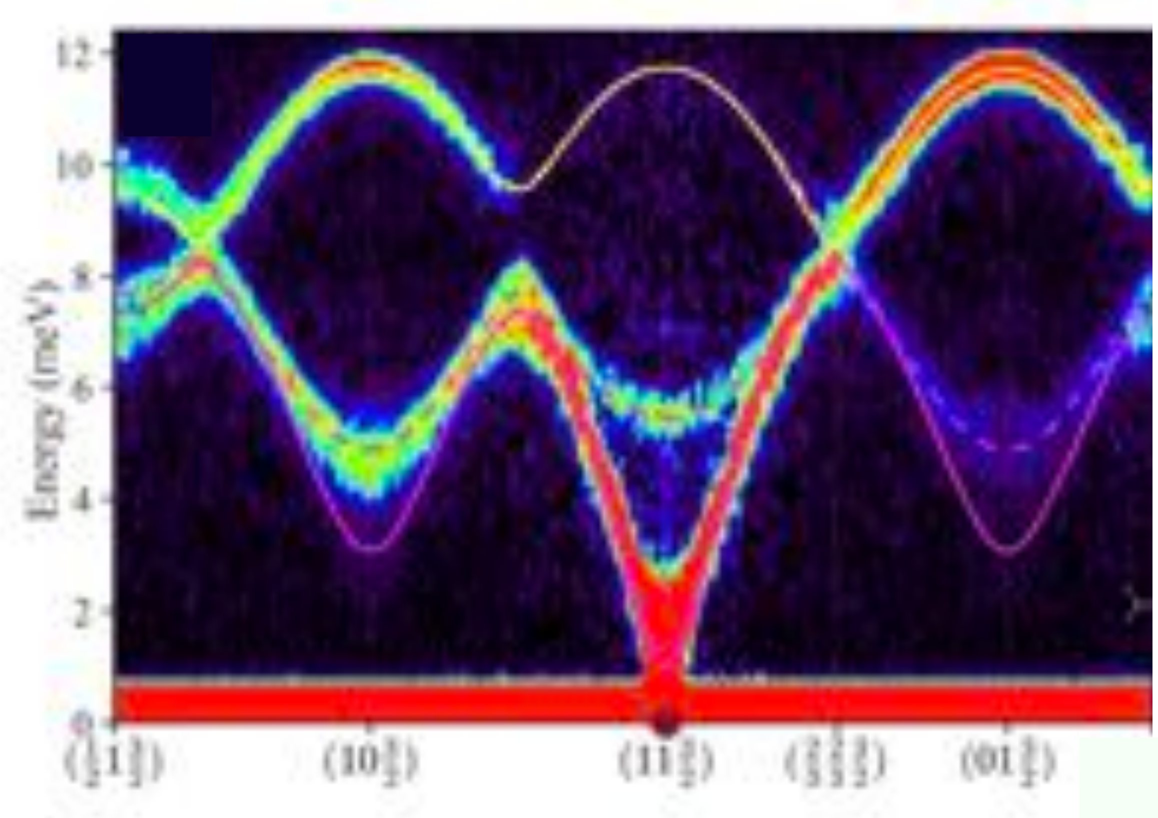


Modern neutron tools



ISIS neutron source: target station 2

LET detector chamber

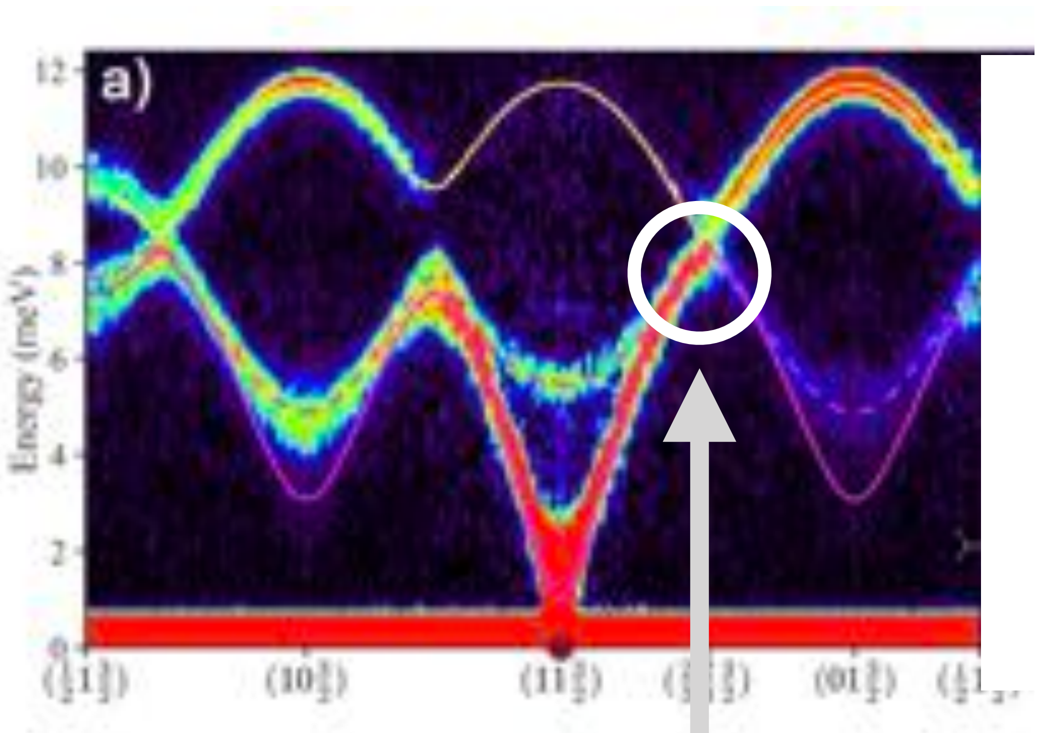


Spin waves in CoTiO3 measured on MERLIN

Three co-aligned single crystals for INS experiment

Intensity variation around linearly dispersing magnons

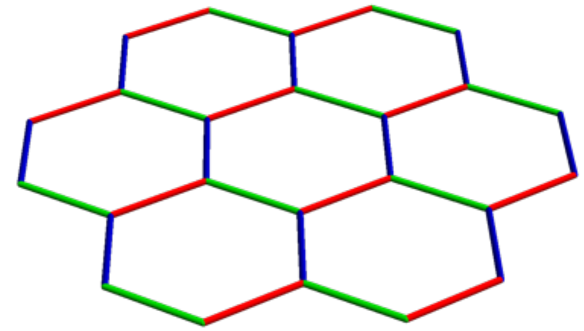
e.g. Nodal Lines

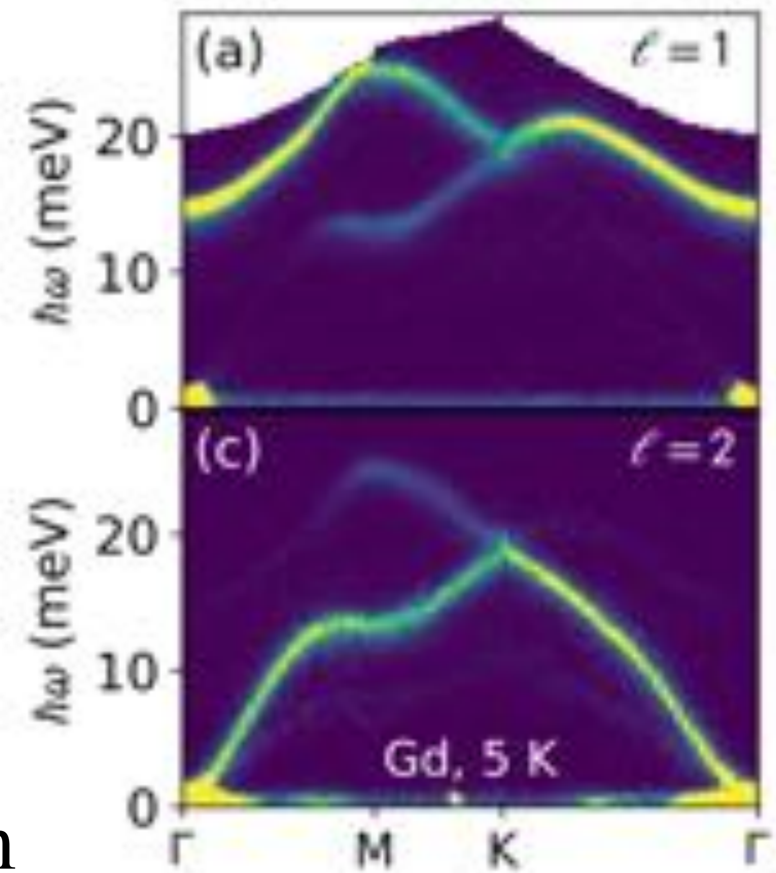


$CoTiO_3$

Dirac point

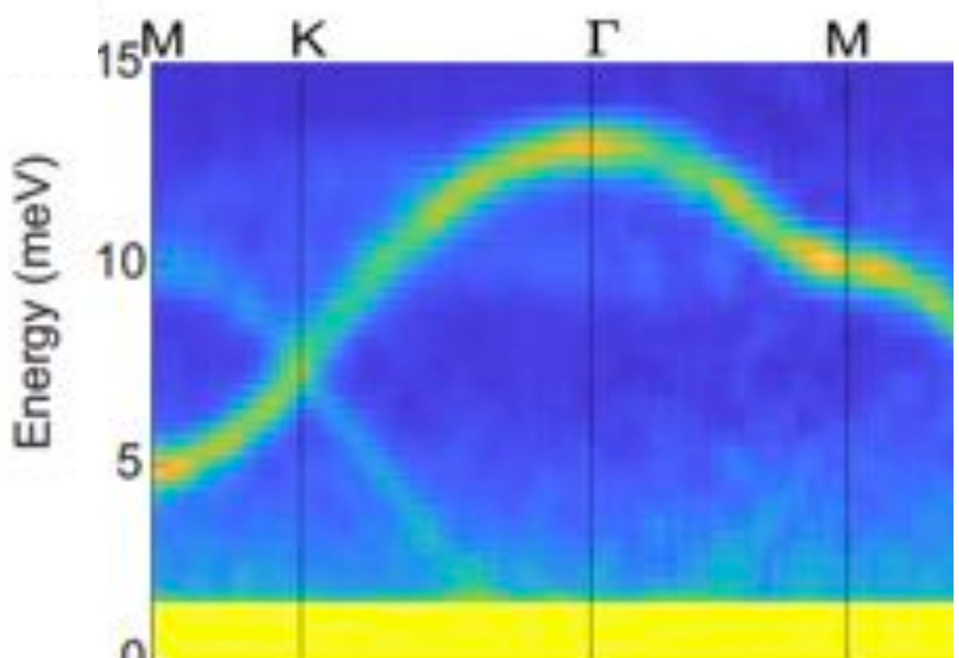
Yuan et al, PRX (2020); Elliot et al. Nat Comm. (2021)



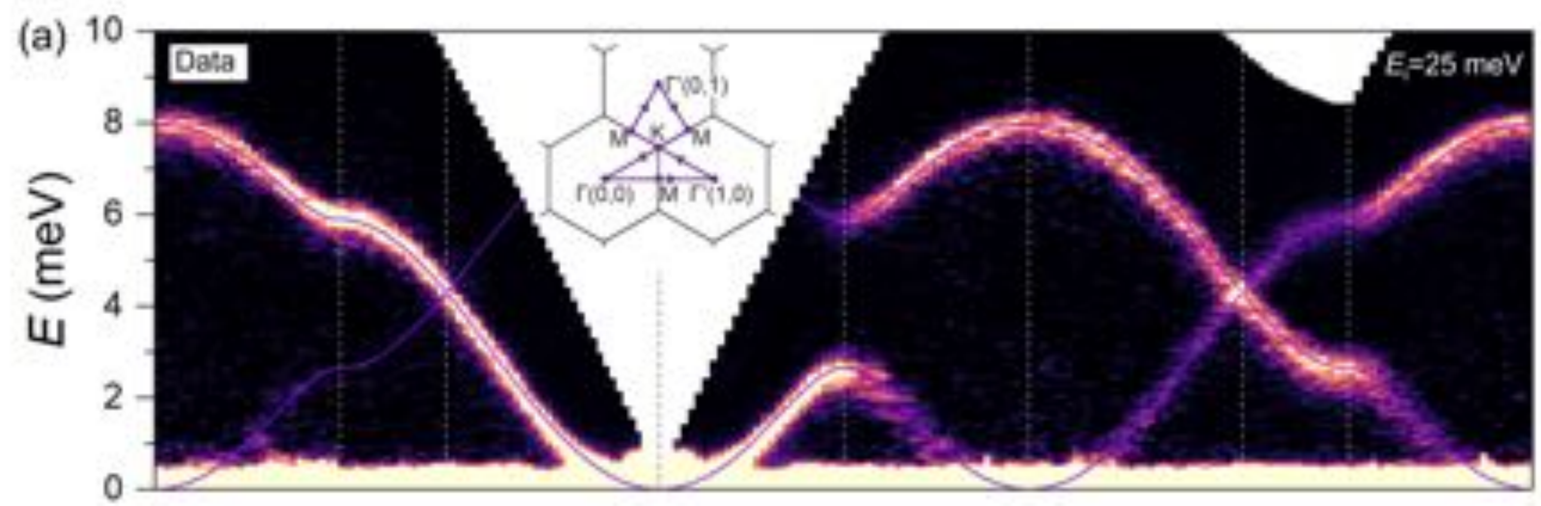


Gadolinium

Scheie et al, PRL (2022)



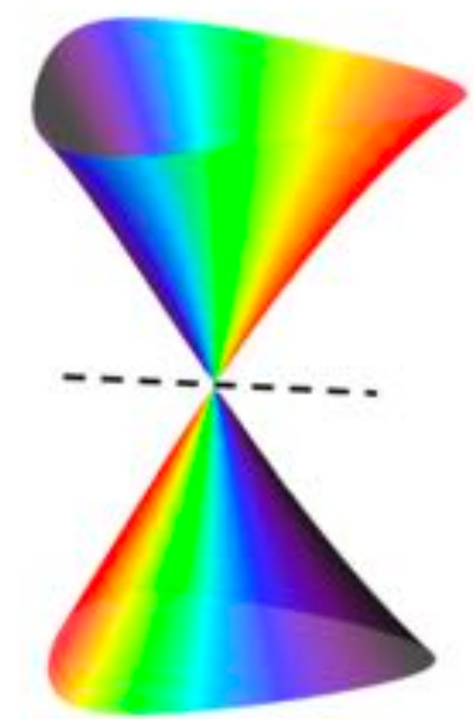
$CrBr_3$ Nikitin et al. (2022)



$CrCl_3$ Seung-Hwan Do et al. (2022)

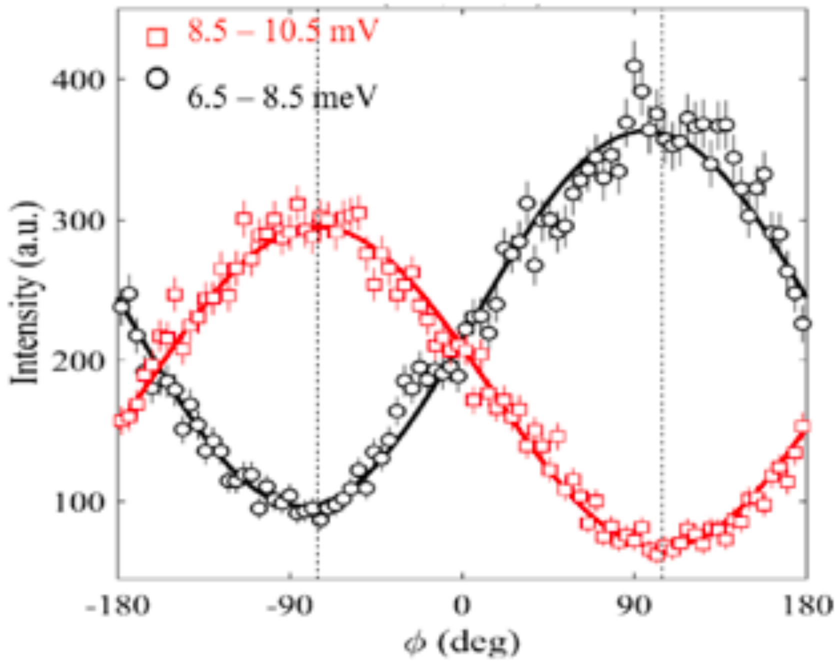
A Winding Number in Magnons

Inelastic neutron scattering signature of spin momentum locking



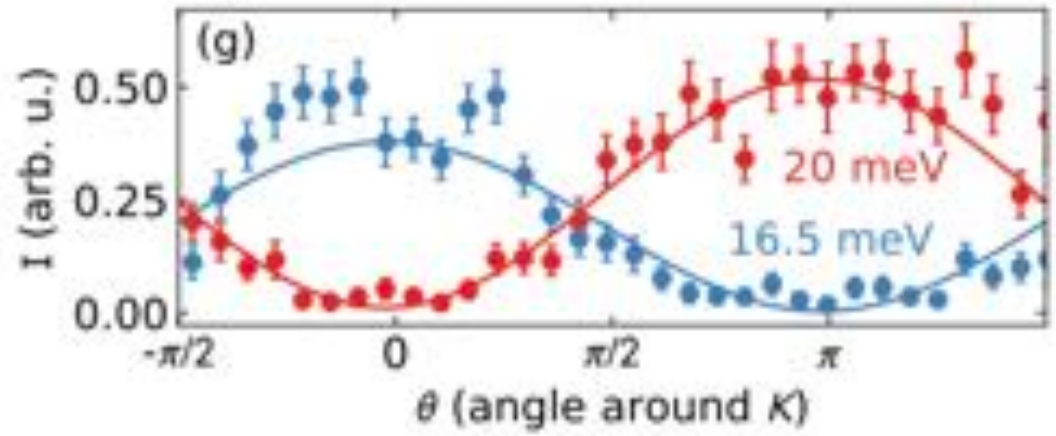
Shivam, Moessner, Coldea, PM (2017)

$$H_{\text{eff}} = v\boldsymbol{k}\cdot\boldsymbol{\sigma}$$



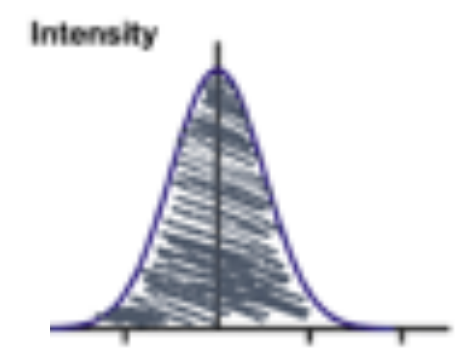
$CoTiO_3$

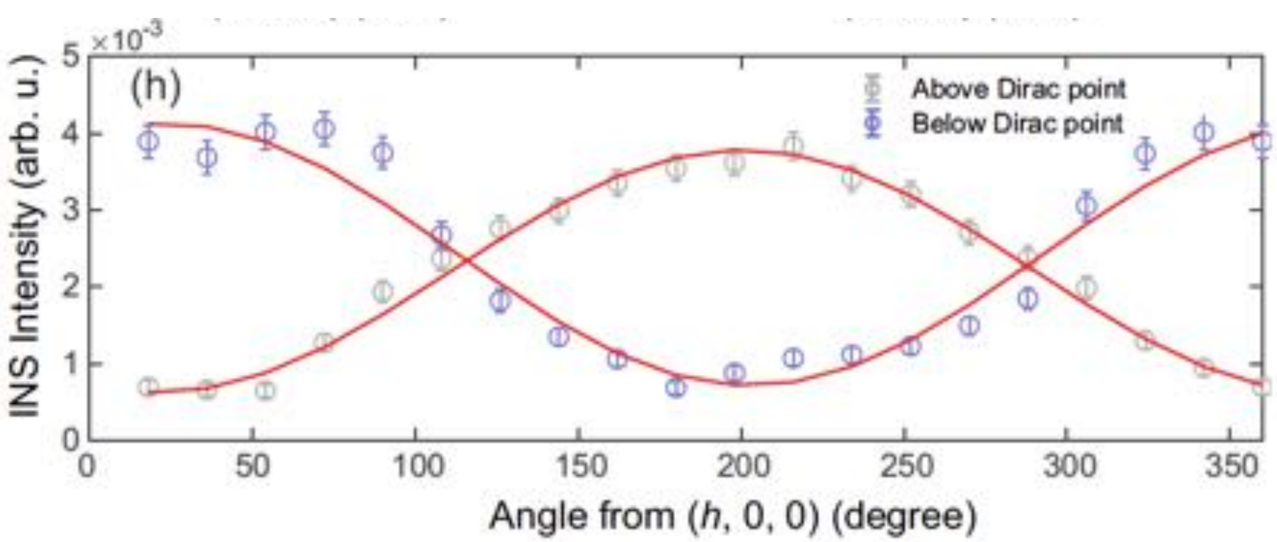
Elliot, PM et al. Nat Comm. (2021)



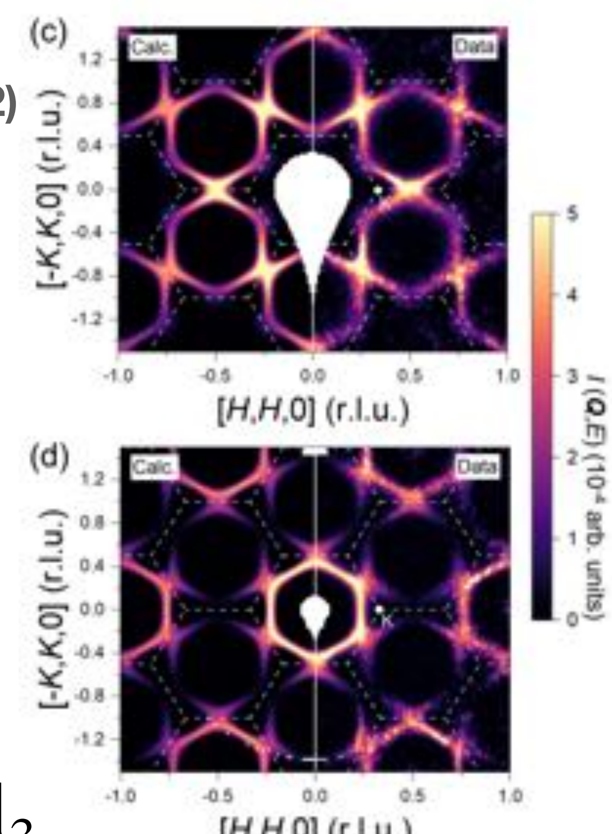
Gadolinium

Scheie, PM et al, PRL (2022)





$CrBr_3$ Nikitin et al. (2022)

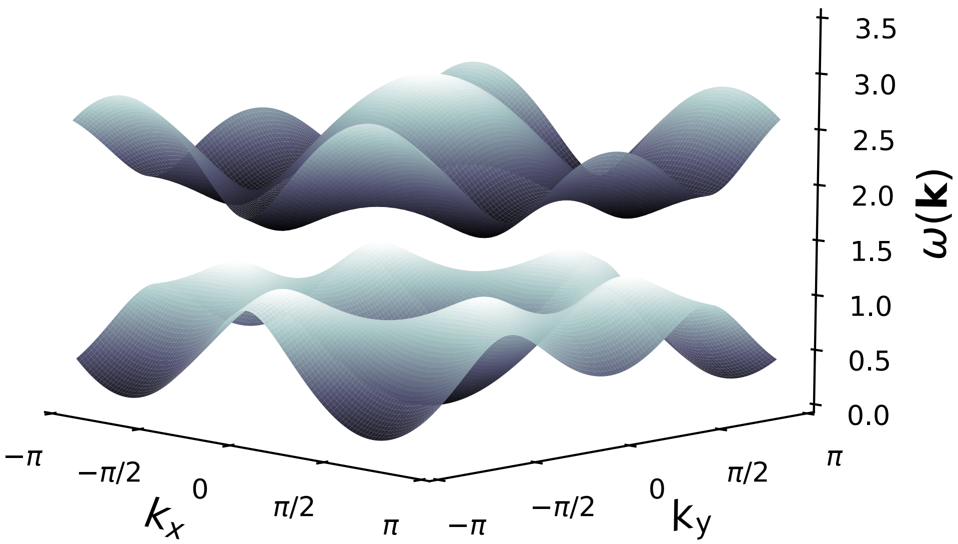


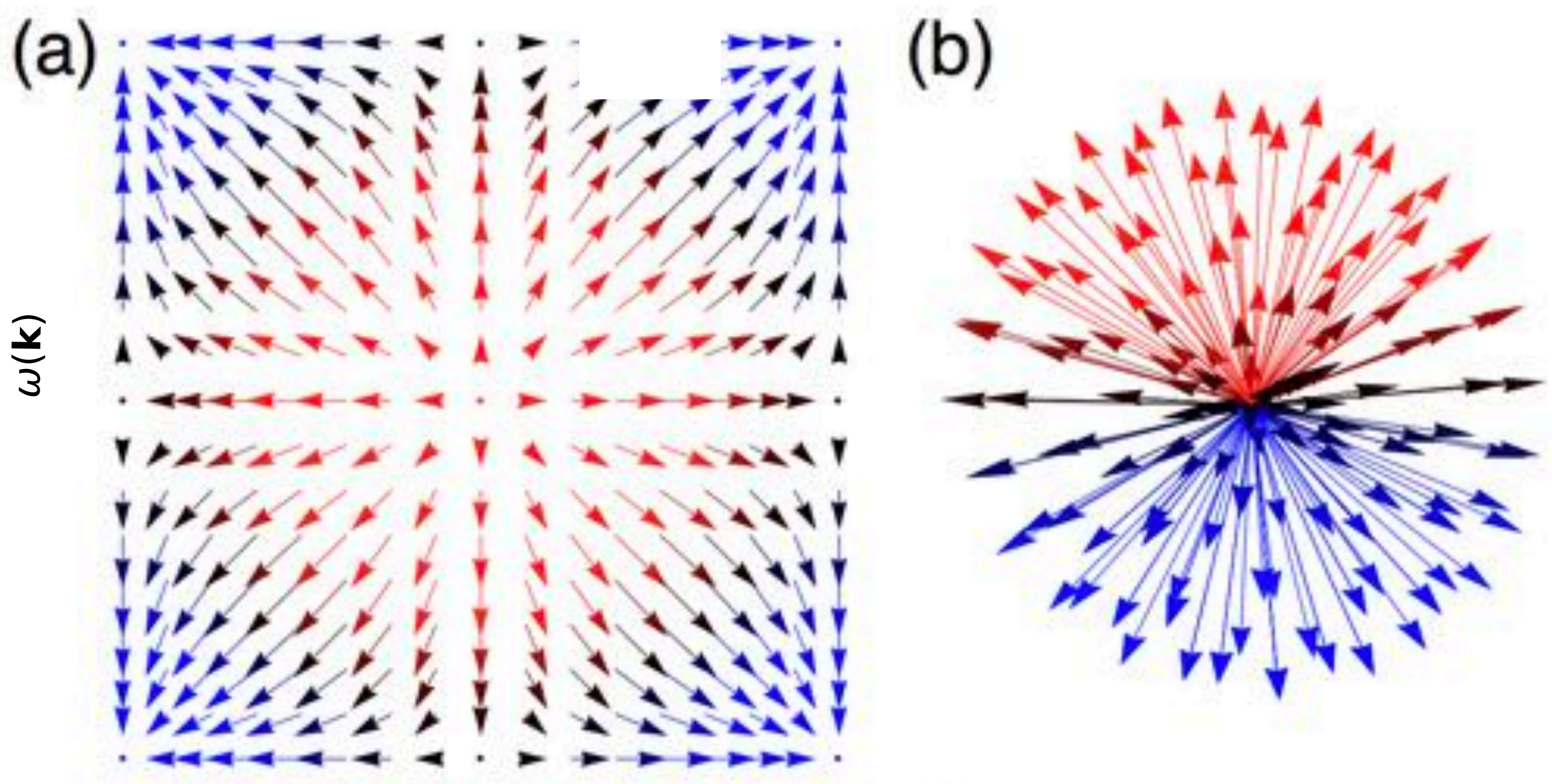
$CrCl_3$

Seung-Hwan Do et al. (2022)

Topology

Wavefunctions in Brillouin zone may carry winding numbers that are insensitive to local deformation of the band structure





Bulk topological invariants have observable consequences.

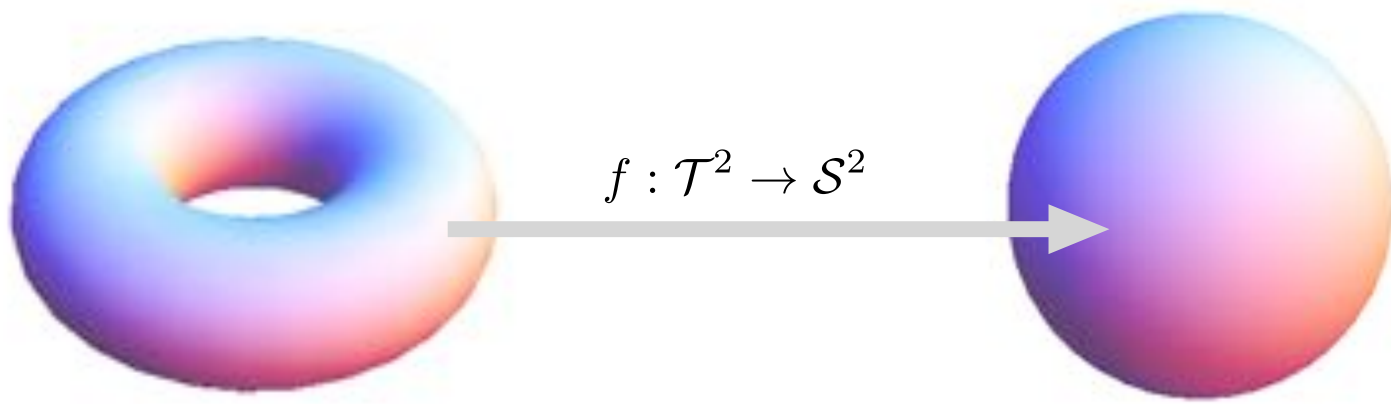
Hermitian topology: Bulk-boundary correspondence tells us that topological invariant is associated with presence of surface states

Topological Magnons

Winding numbers in wavefunctions with observable consequences

- Simple example: Chern number in two band model

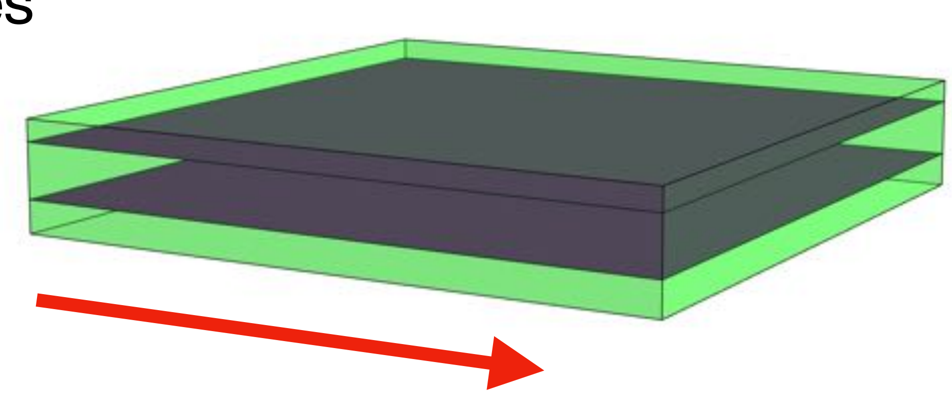
$$H(k_x, k_y) = \boldsymbol{d}(k_x, k_y) \cdot \boldsymbol{\sigma}$$



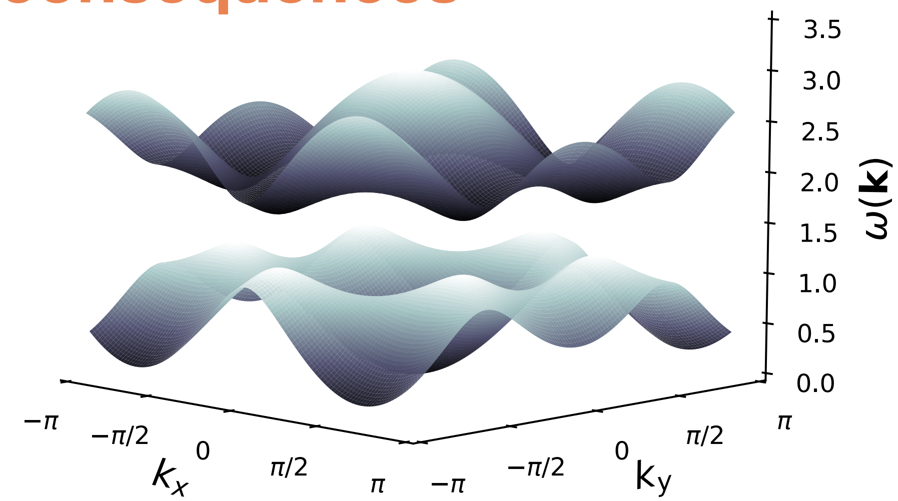
$$\nu = \frac{1}{4\pi} \int_{\mathbb{T}^2} d^2\boldsymbol{k}\, \hat{\boldsymbol{d}} \cdot \left(\frac{\partial \hat{\boldsymbol{d}}}{\partial k_x} \times \frac{\partial \hat{\boldsymbol{d}}}{\partial k_y} \right)$$

- Chern number: sphere covering number

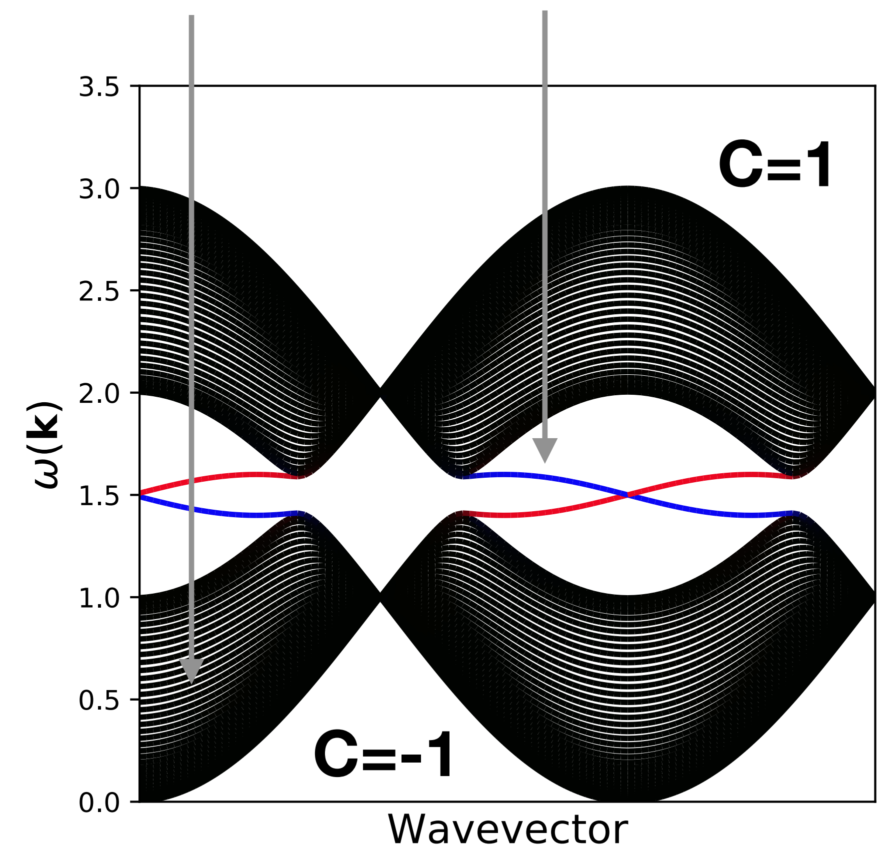
- Chiral surface states



Edge state group velocity



Bulk bands Surface state

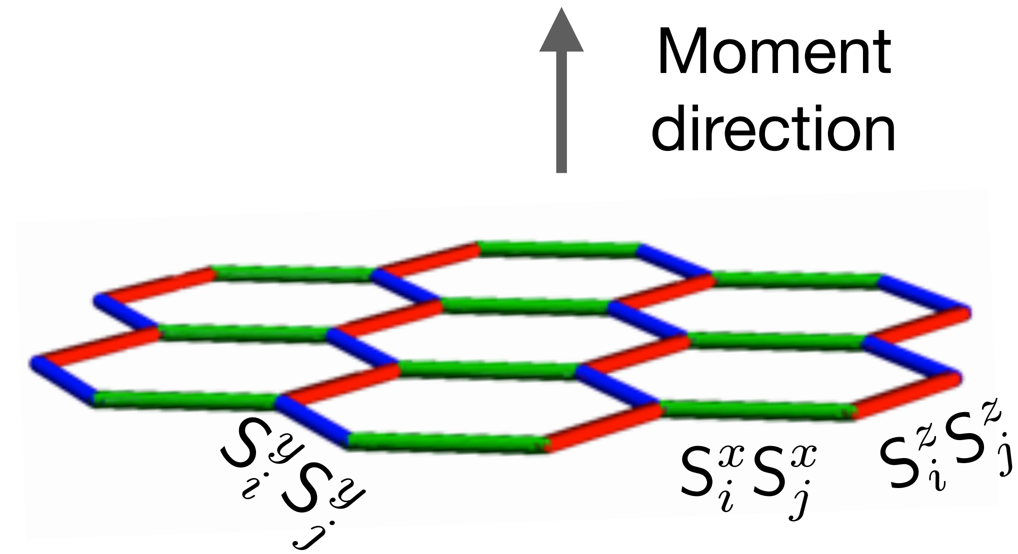


Spectrum on slab geometry

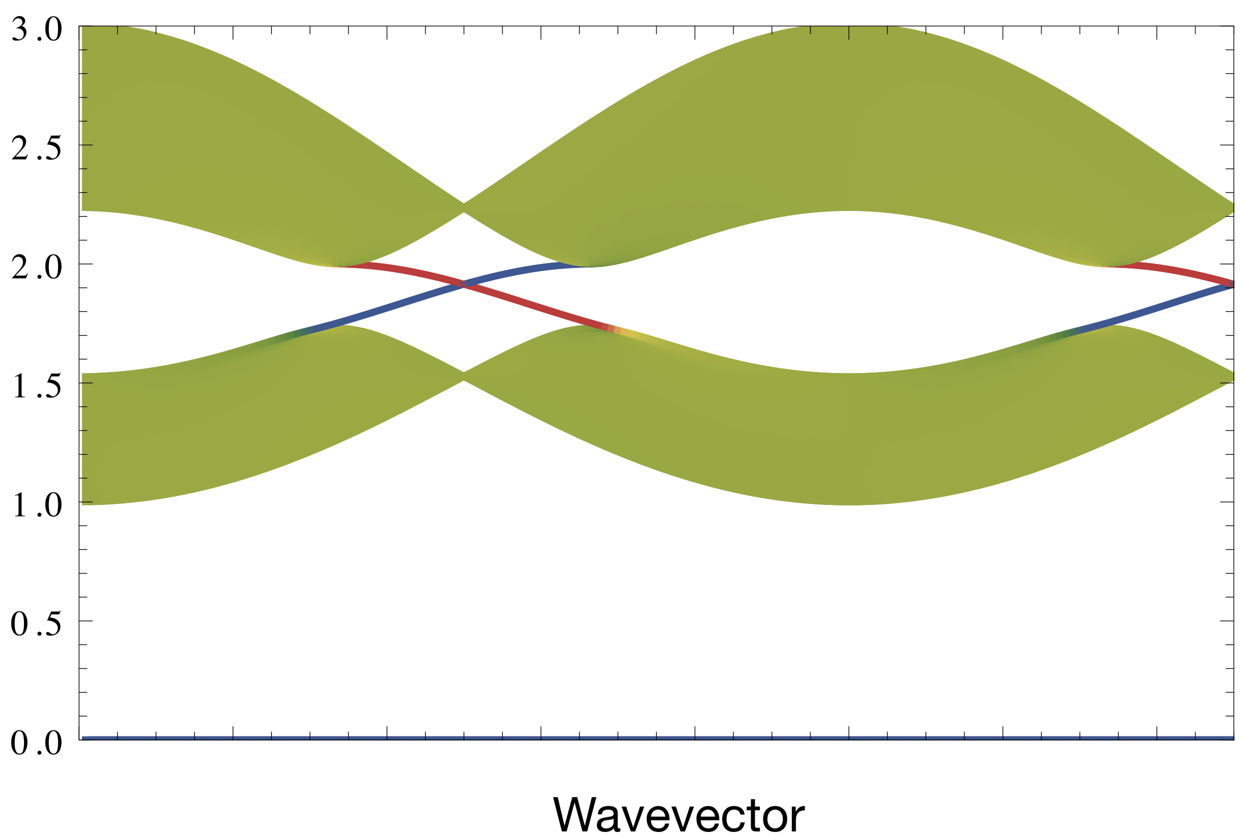
Chern Magnon Bands: An Example

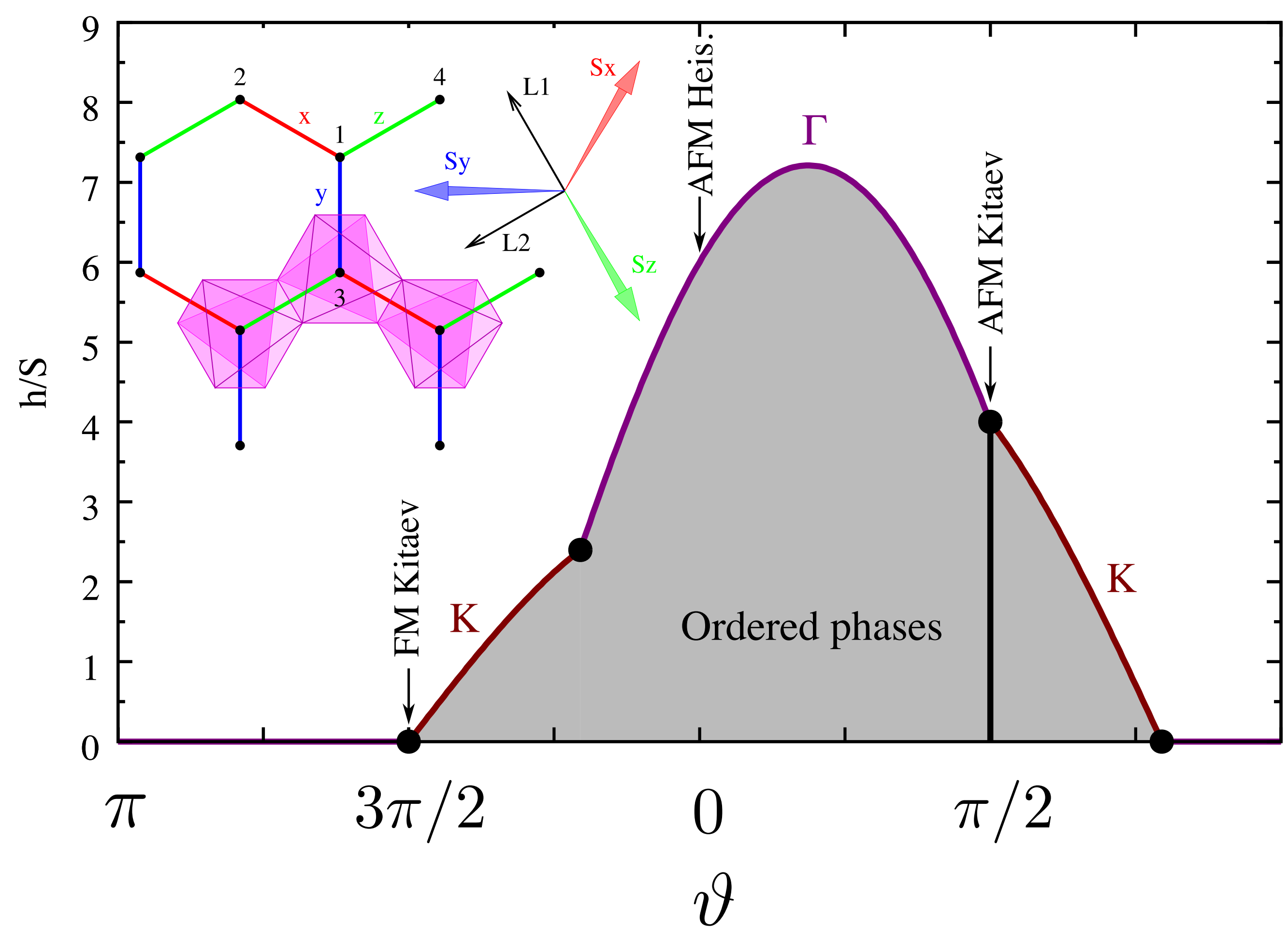
Kitaev-Heisenberg Model

$$\mathcal{H} = J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + \sum_{\langle i,j \rangle_\gamma} 2K S_i^\gamma S_j^\gamma - \boldsymbol{h} \cdot \sum_i \mathbf{S}_i$$



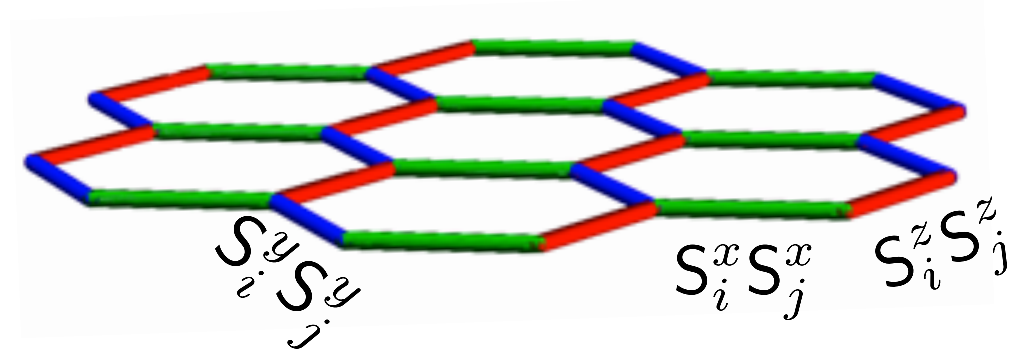
- Relevant to various honeycomb ruthenates, iridates …
- Topological bands present across phase diagram in polarized phase





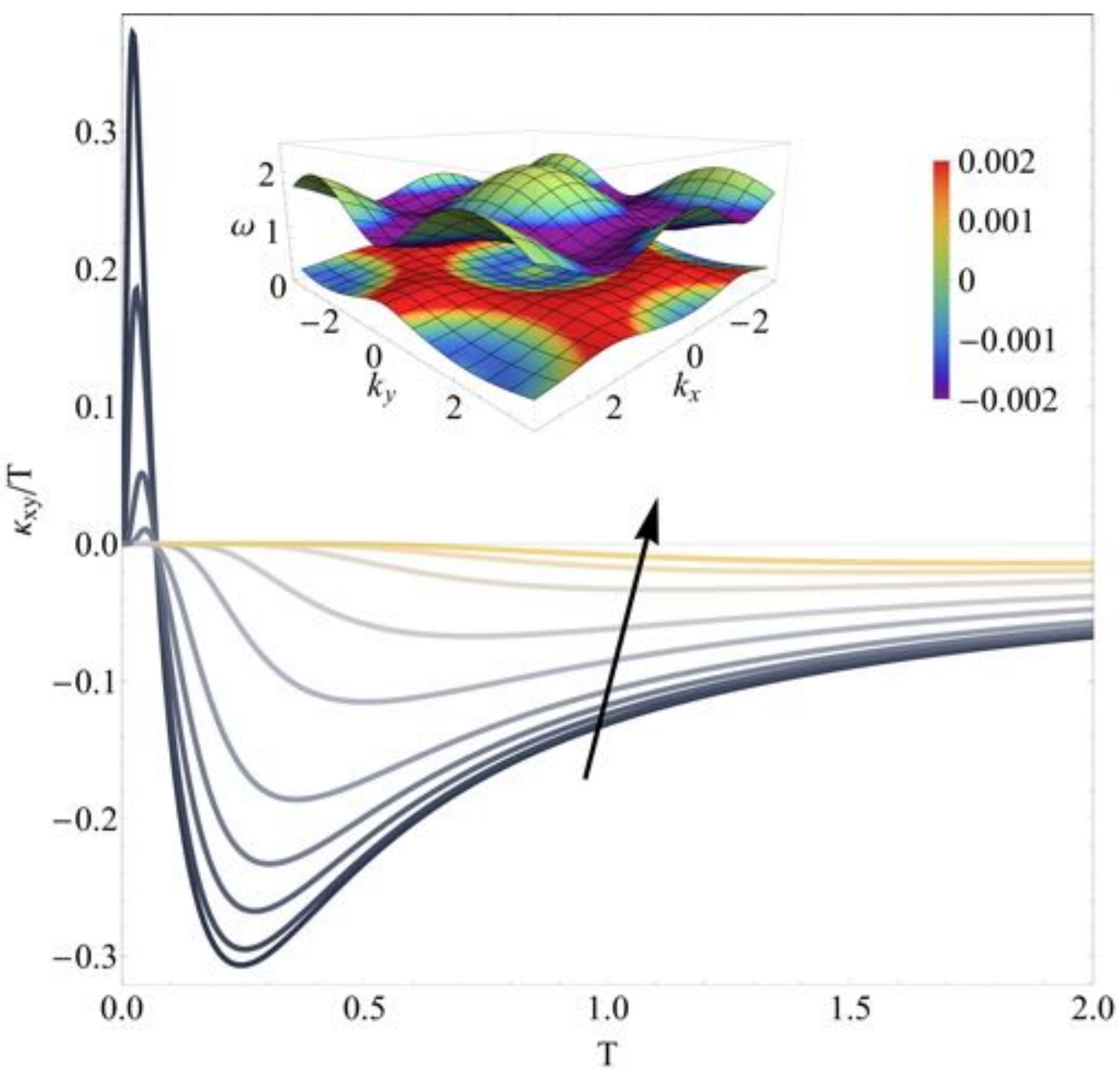
PM, Dong, Gohlke, Rau, Pollmann, Moessner Penc, PRB 98 060404 (2018)

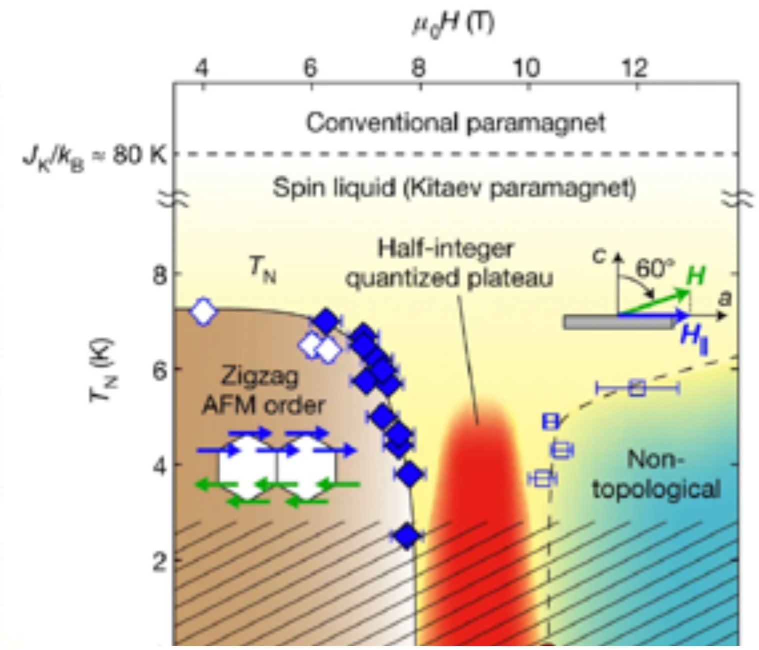
``Gapped" Magnon Topology

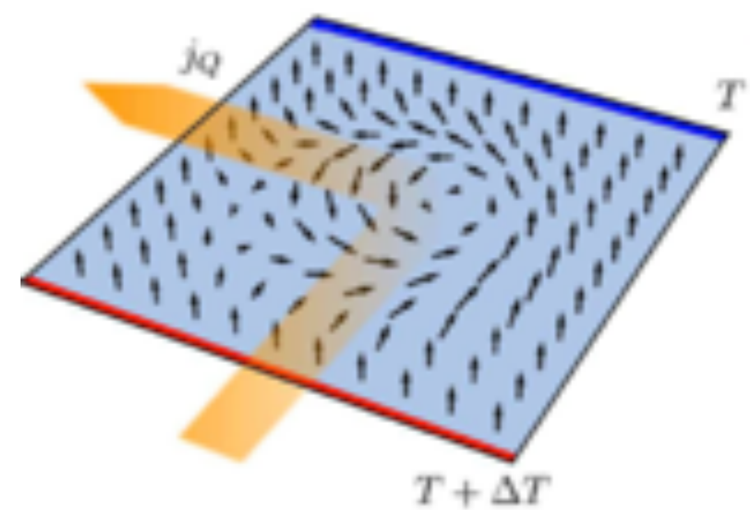


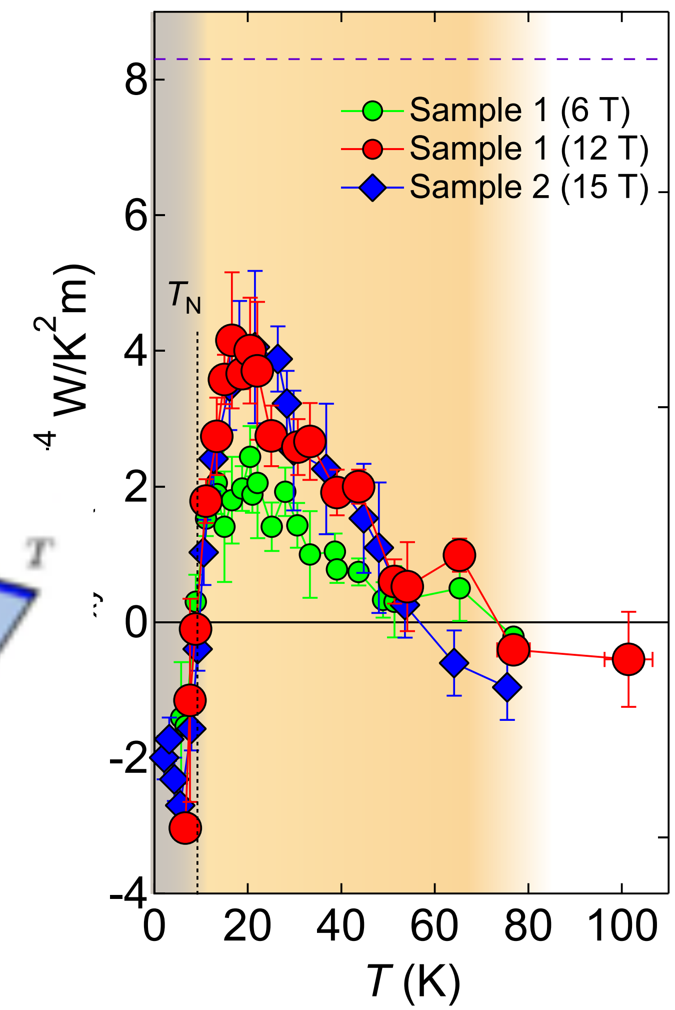
Many Chern magnon models…

e.g. Kitaev-Heisenberg-Gamma model in polarized phase









Berry curvature of Chern bands leads to thermal Hall effect

Kasahara et al. PRL (2018)

Candidate to explain thermal Hall at intermediate fields in $\alpha-RuCl_3$ though controversial

Detection of surface states presents a challenge: neutral, microscopic, low energy

PM, Dong, Gohlke, Rau, Pollmann, Moessner Penc, PRB 98 060404 (2018)

Czajka et al., Naturę Materials 2022

Zhang et al., PRB 2021

Varieties of Topological Magnons - A Snapshot

Chern Magnons - various different ground states

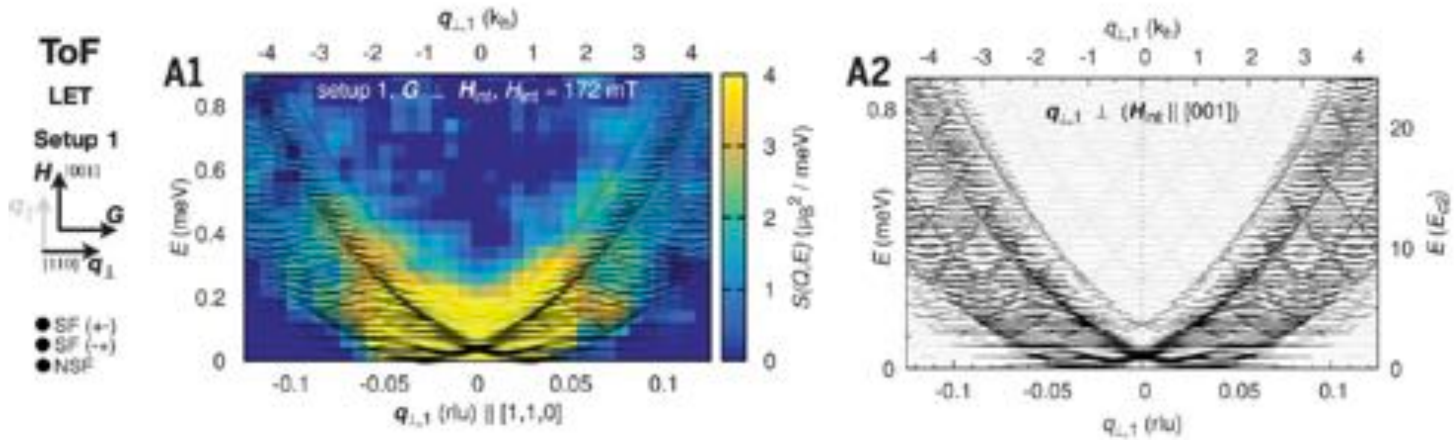
Owerre, JPCM (2016)

Shindou, Matsumoto, Murakami, Ohe, PRB (2012)

Chisnell et al., PRL (2015)

PAM, Dong, Gohlke, Pollmann, Moessner, Penc, PRB (2018)

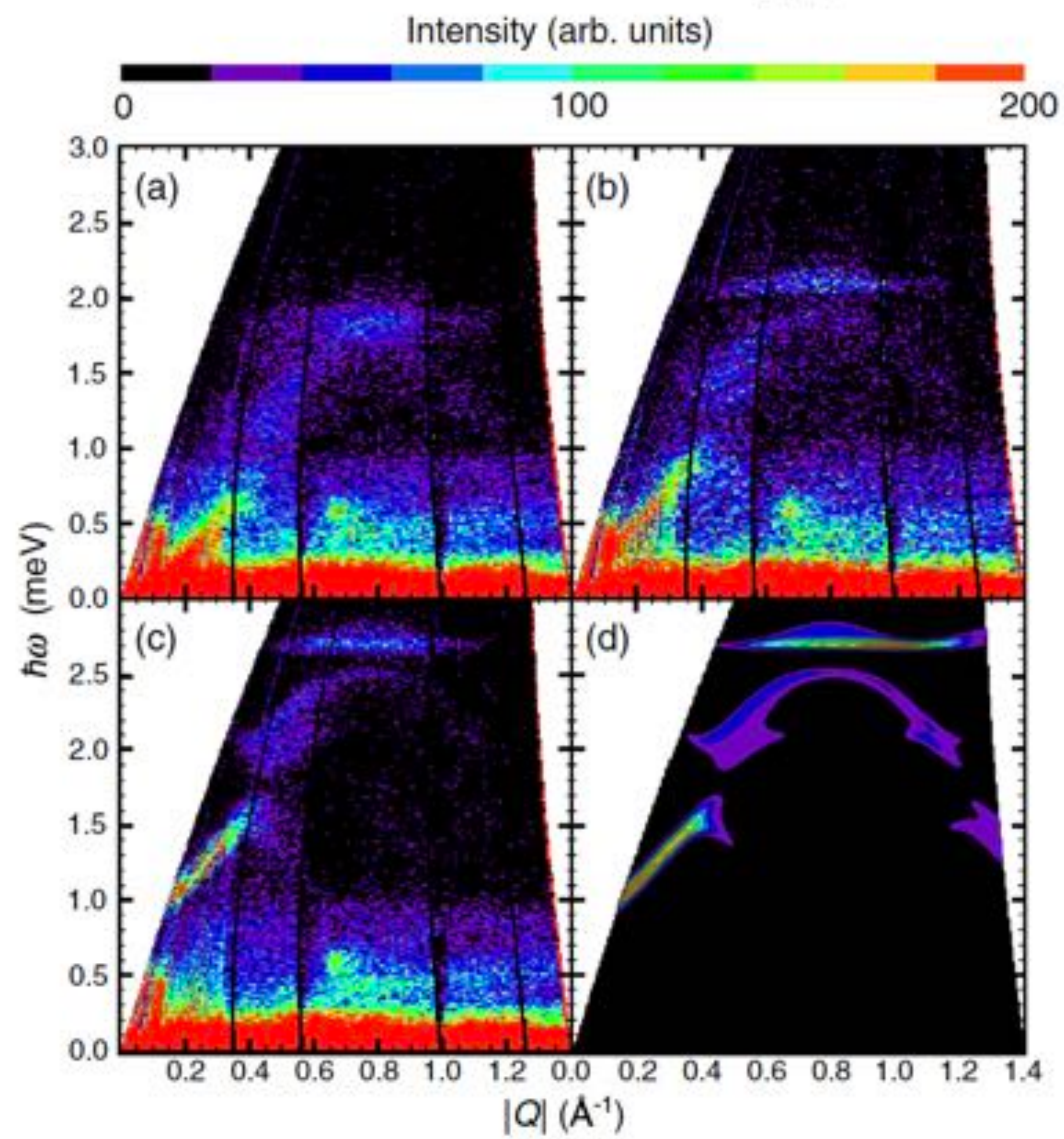
Joshi, PRB (2016)



Weyl Magnons

Li, Li, Kim, Balents, Yu, Chen Nature Comm. (2016)

Jian, Nie, PRB (2018)



Dirac Magnons

Yuan et al., PRX (2020)

Scheie et al., PRL (2022)

Higher Order Degeneracies

Corticelli, Moessner, McClarty, PRL (2023)

Antiferromagnetic TI

Kondo, Akagi, Katsura (2019)

Magnon Landau Levels

Weber et al., Science (2022)

Higher Order TIs

YB Kim et al. PRB (2021) and Mook et al., PRB (2021)

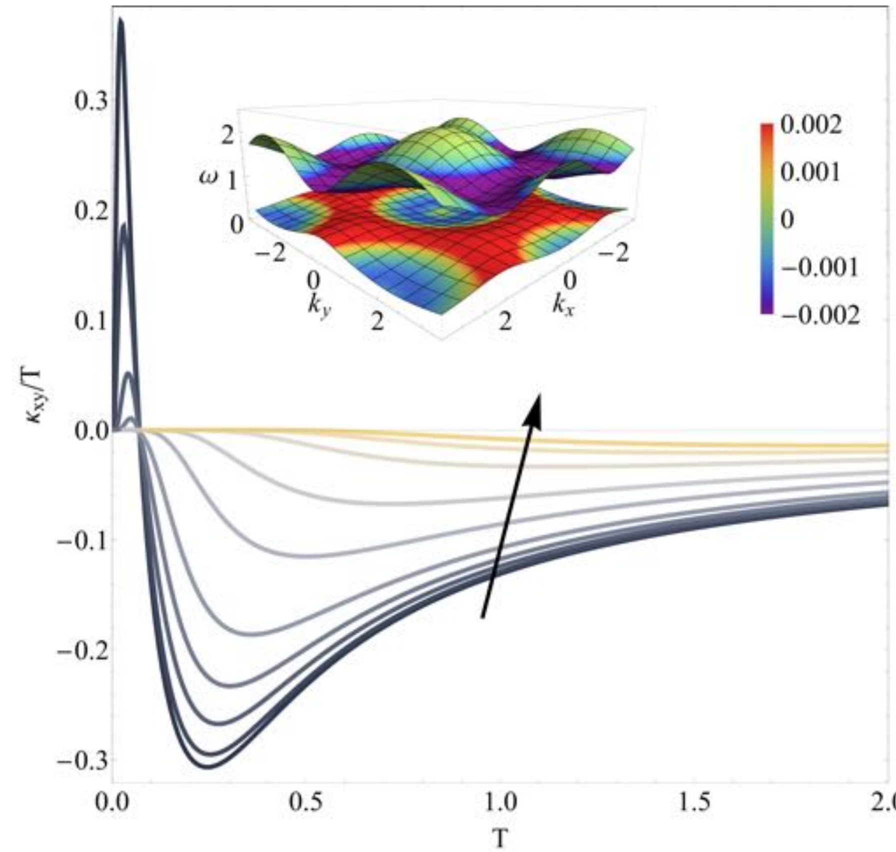
Review: PM, Annual Rev. Condensed Matter Physics (2022)

Main Differences between Electronic and Magnonic Topological Materials

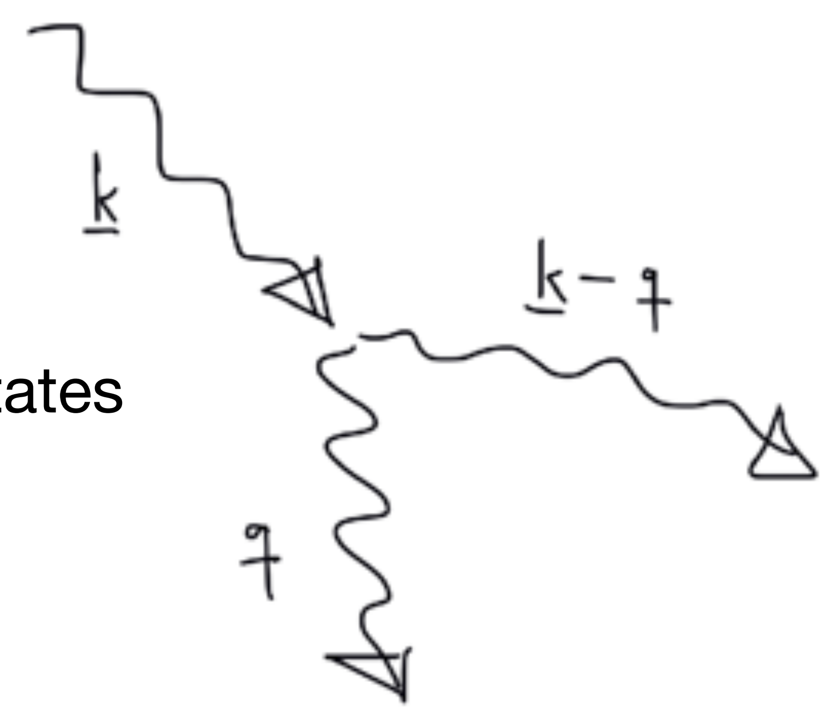
Bosonic and at finite energy

Generally no quantized response

But instead varied set of response functions in magnon systems including thermal Hall for Chern magnon bands



Bulk-boundary correspondence and magnonic surface states



Interactions always present - these may be crucial

How to measure surface states

General problem of how to probe single magnetic layers

e.g. van der Waals magnets

For structure can use X-rays but harder for dynamics/excitations

Neutrons interact weakly with matter : bulk probe only

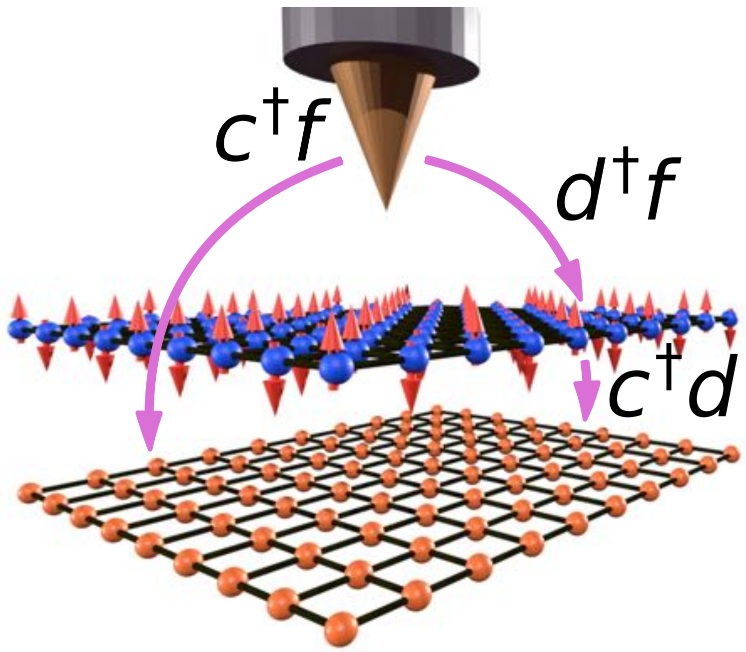
Magnetization, heat capacity etc. : hard to discern signal

...though for mesoscopic systems may use Hall measurements for magnetization

Two proposals so far:

Pump into surface states

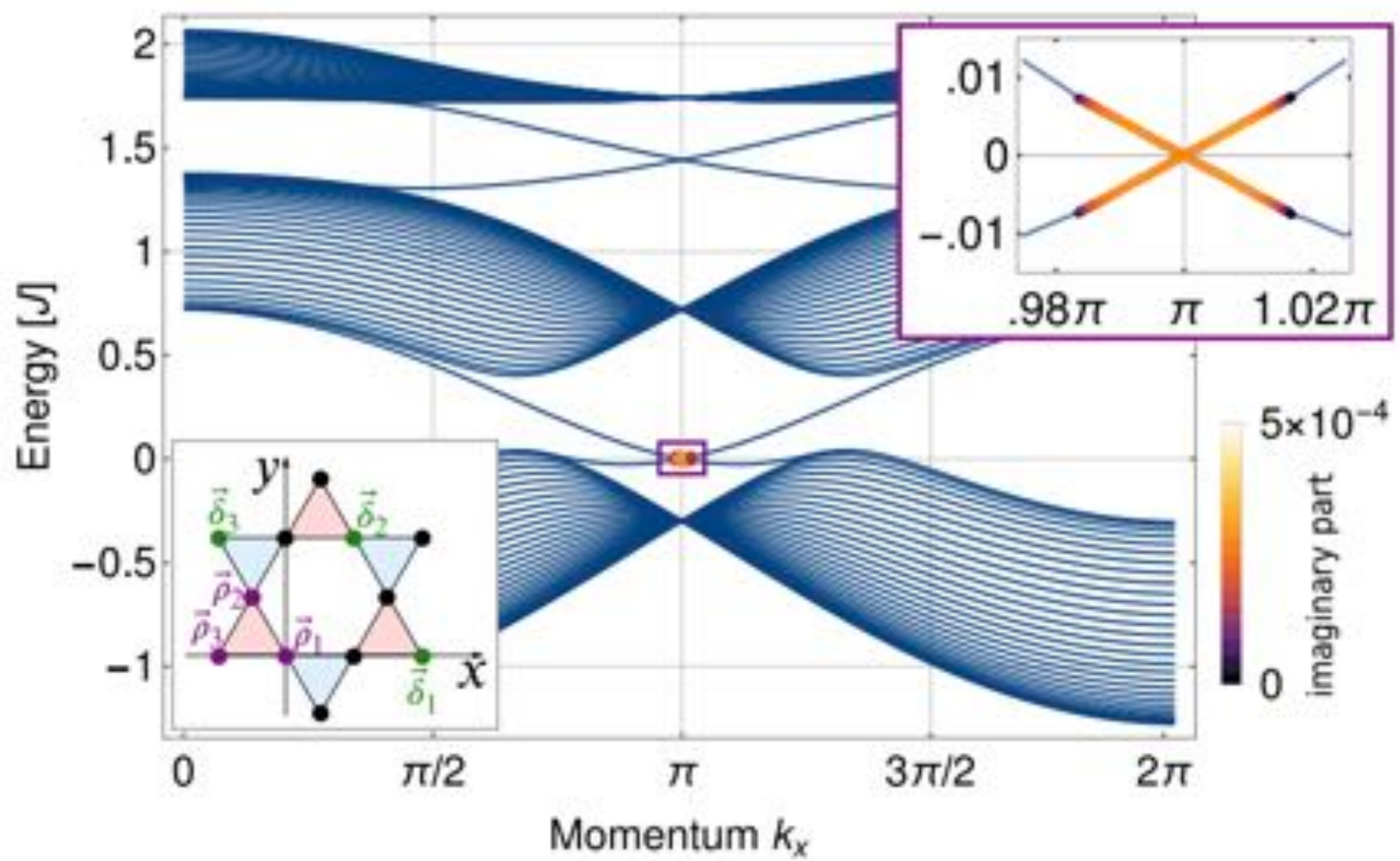
Electronic tunneling



Topological Magnon Amplification

Radiation field coupling to surface states - in particular the anomalous terms

$$\sum \frac{g_k}{2}\left(a^\dagger_{-k}a^\dagger_k b + \mathbf{h.c.}\right)$$

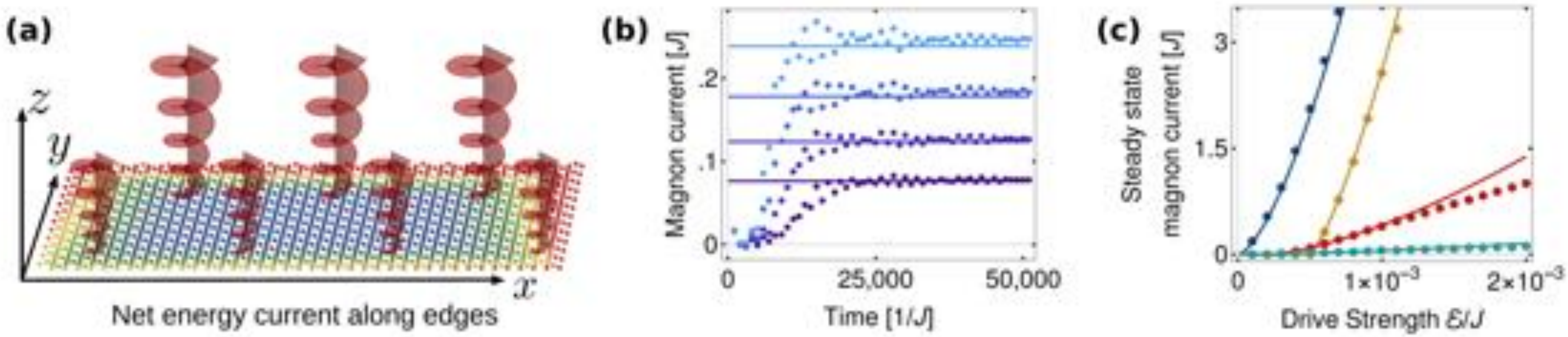


Kagome ferromagnet with out-of-plane DMI has Chern magnon bands

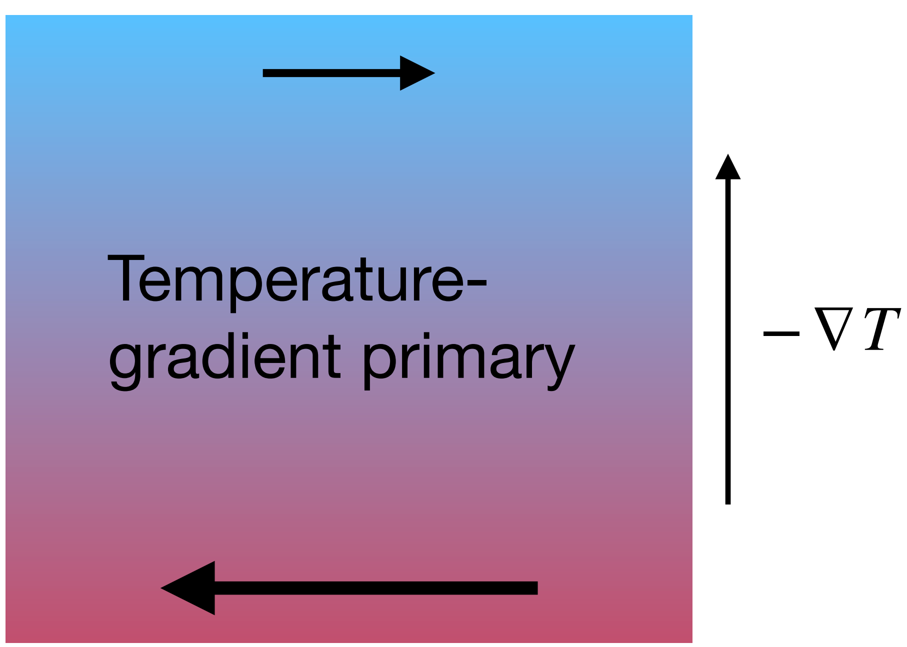
For certain edge states couple $k = \pm\,\pi$ modes

Malz, Knolle, Nunnenkamp, Nature Comm. (2019)

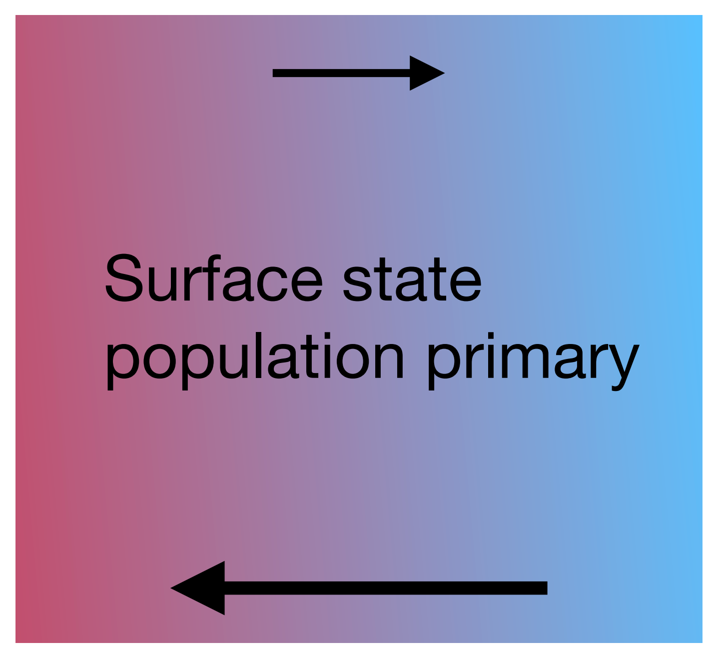
Topological Magnon Amplification II



Conventional thermal Hall effect



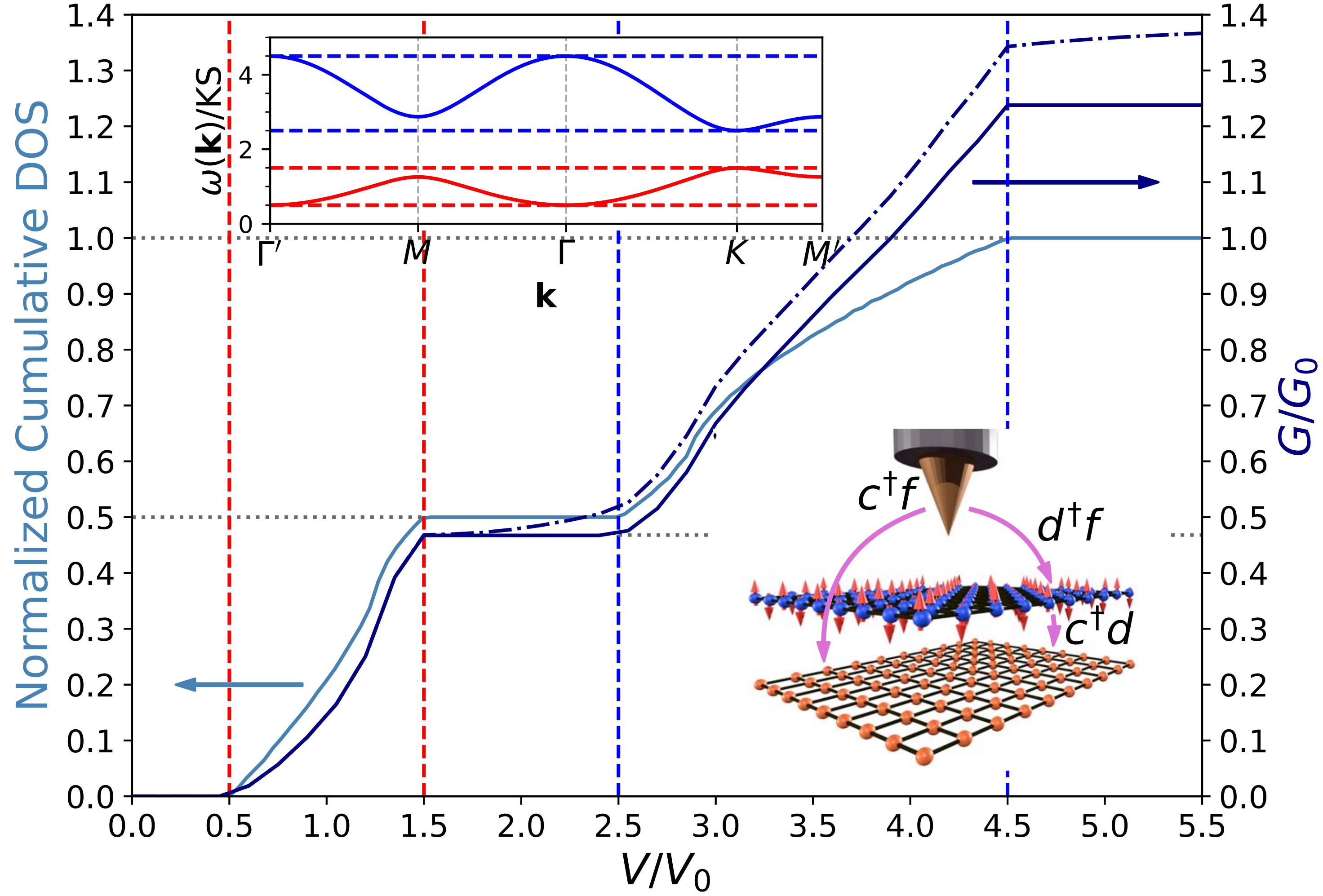
Driven case



Malz, Knolle, Nunnenkamp, Nature Comm. (2019)

Inelastic Tunneling

Single site case: pick up (approximately) local density of states

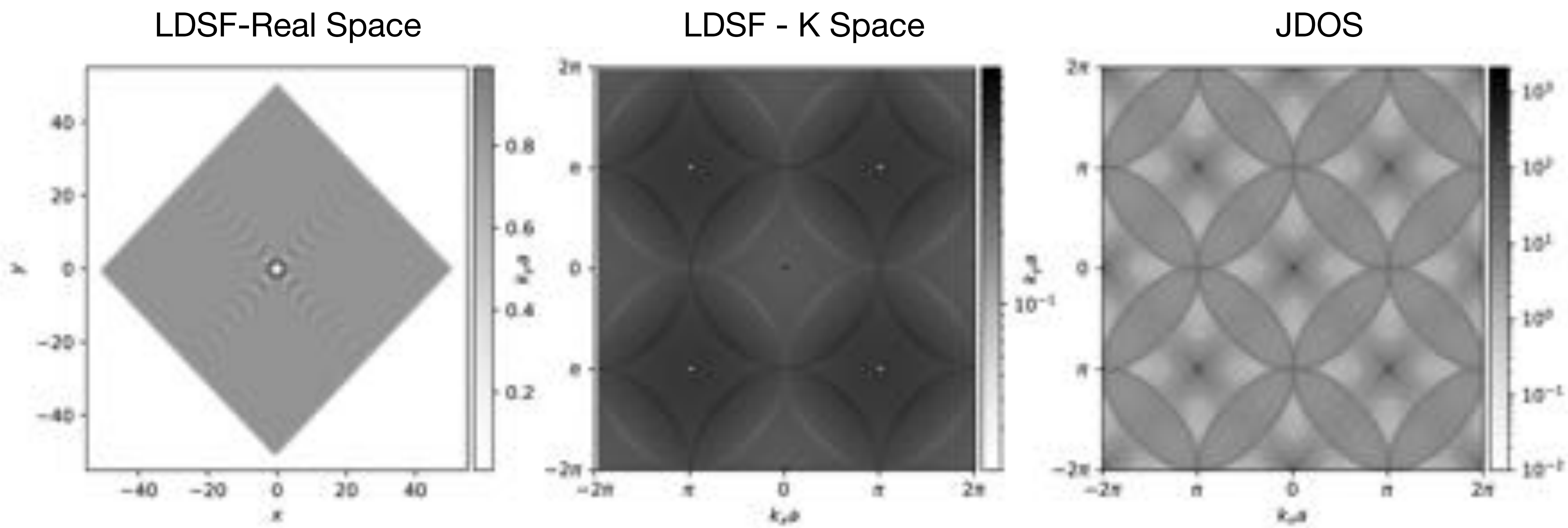


See also work from Knolle et al. for density of magnetic states

Quasi-particle Interference

Scattering of magnons from “simple” disorder leads to clear interference patterns

e.g. single vacancy - a non-magnetic ion in magnetic lattice



This interference pattern is connected to the joint density of states

$$\mathcal{J}(\omega, \boldsymbol{q}) = \sum_{\boldsymbol{k}} \delta_{\omega,\epsilon_{k+q}} \delta_{\omega,\epsilon_k}$$

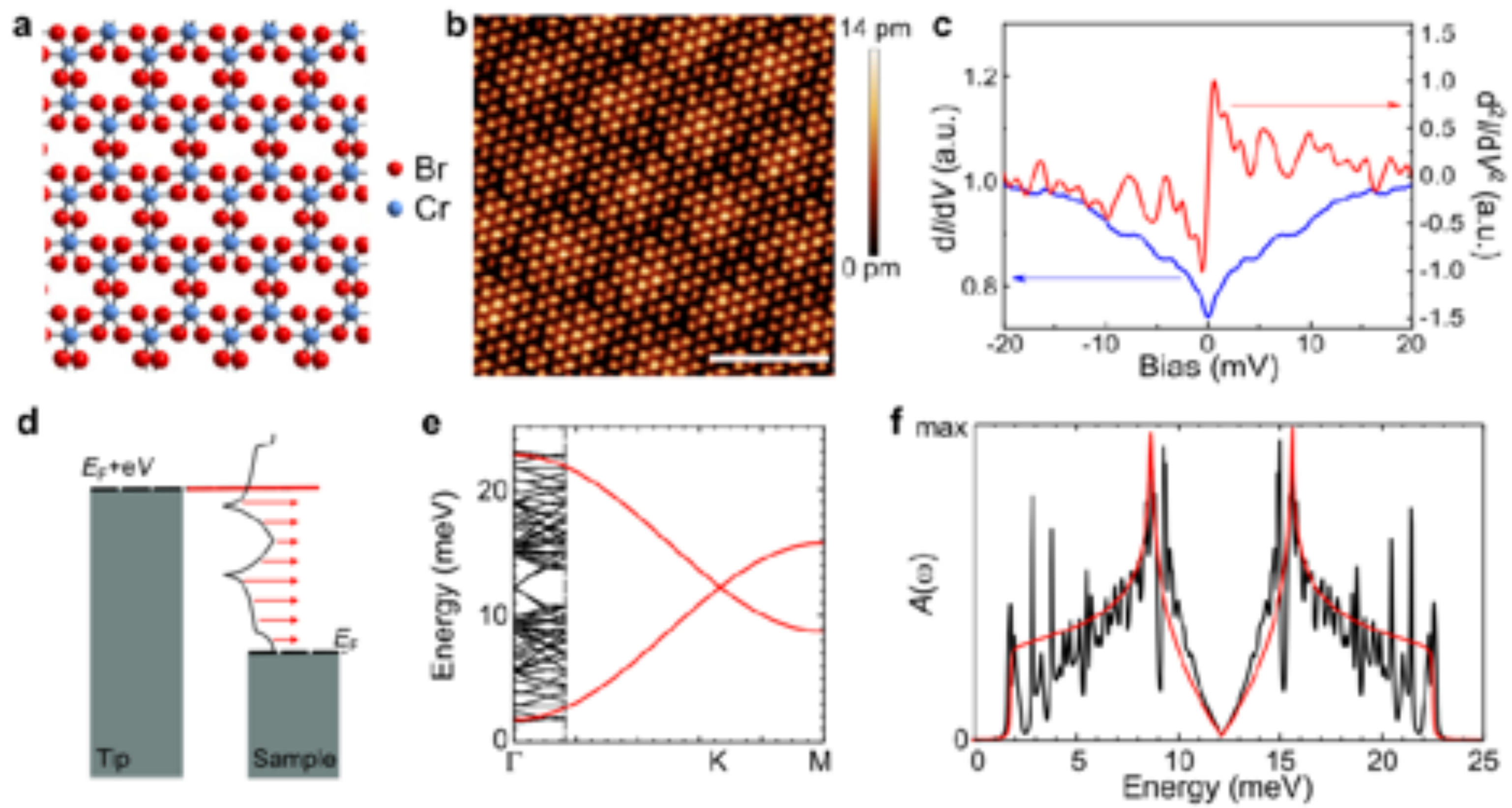
Mitra, Corticelli, Ribeiro, PM, PRL (2023)

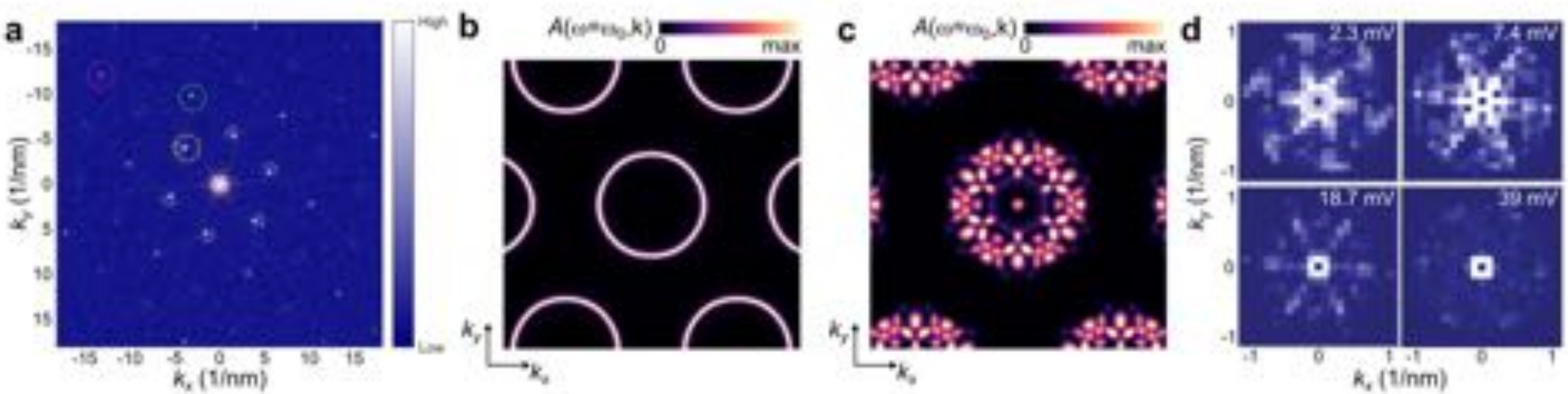
Experimental progress

Work from Somesh Chandra Ganguli, Markus Aapro, Shawulienu Kezilebieke, Mohammad Amini, Jose L. Lado, Peter Liljeroth

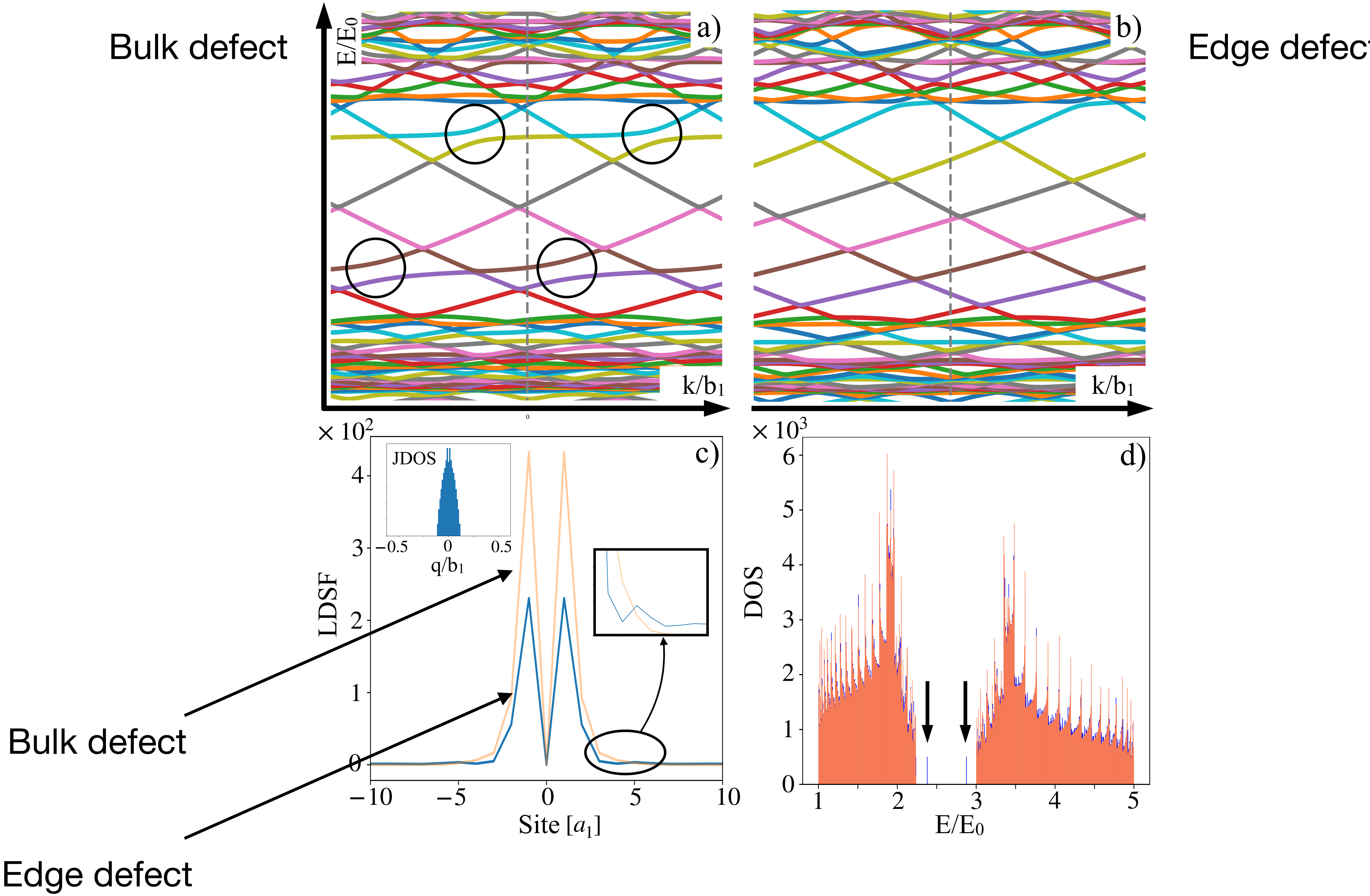
Nano letters (2023)

Quasi-particle interference study of $CrBr_3$ monolayer ferromagnet





Magnon Surface States



Mitra, Corticelli, Ribeiro, PM, PRL (2023)

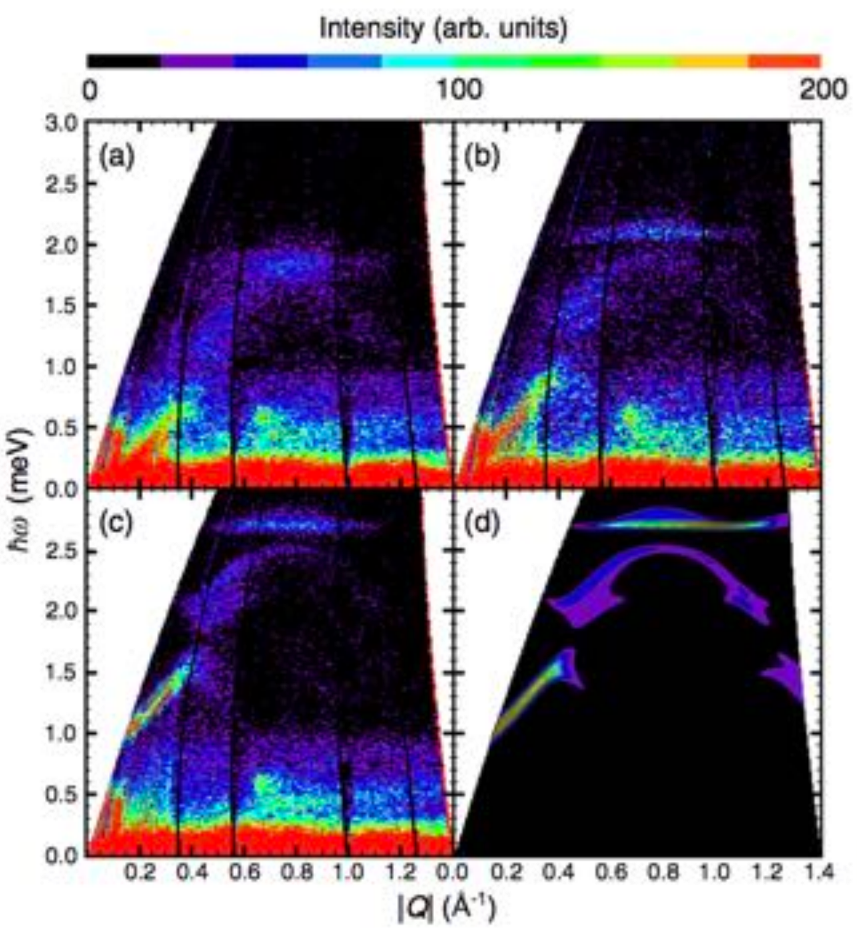
Interplay between topology and interactions

Chern band protection

Interaction-induced topology

Non-Hermitian topology

Topological bound states



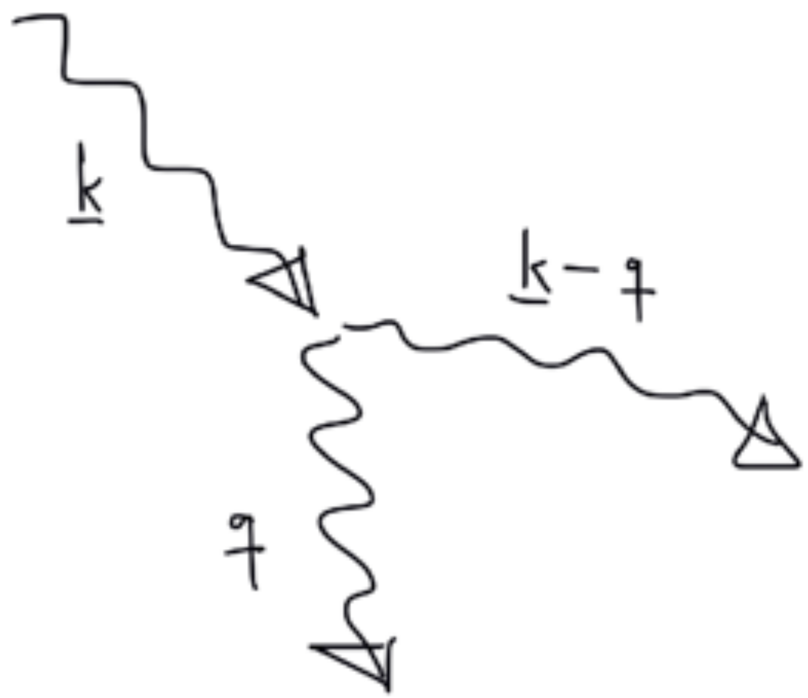
Magnon-Magnon Interactions

Magnon-magnon interactions from Holstein-Primakoff beyond 1/S

$$\mathcal{H}_3 = \frac{1}{2}\sum_{\boldsymbol{k}_\mu} V_3(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)(a^\dagger_{\boldsymbol{k}_1} a^\dagger_{\boldsymbol{k}_2} a_{\boldsymbol{k}_3} + \text{h.c.}) + \ldots$$

Generally number non-conserving terms

Single particle picture may not survive in any detail

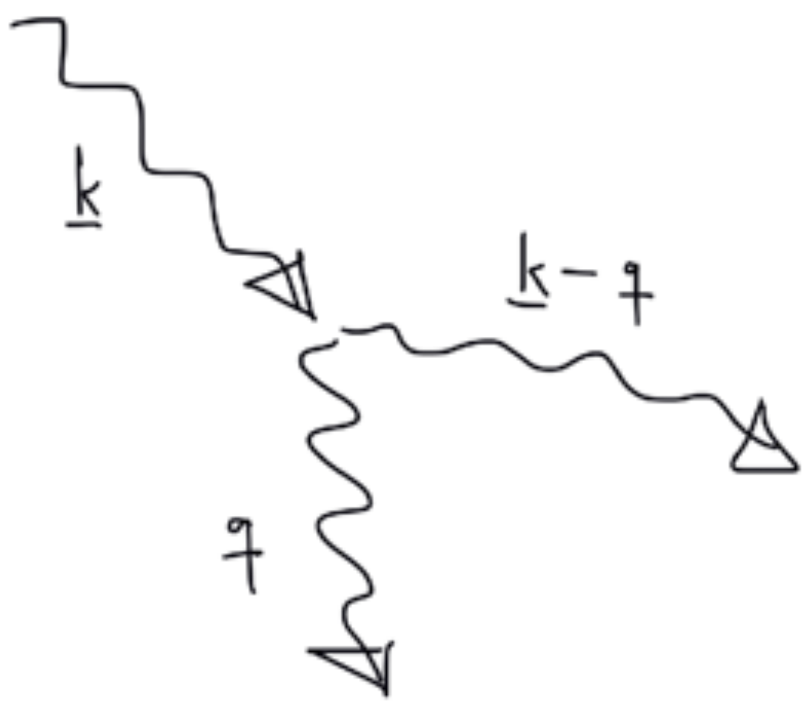


$$\epsilon_{\boldsymbol{k}_3} = \epsilon_{\boldsymbol{k}_2} + \epsilon_{\boldsymbol{k}_1}$$

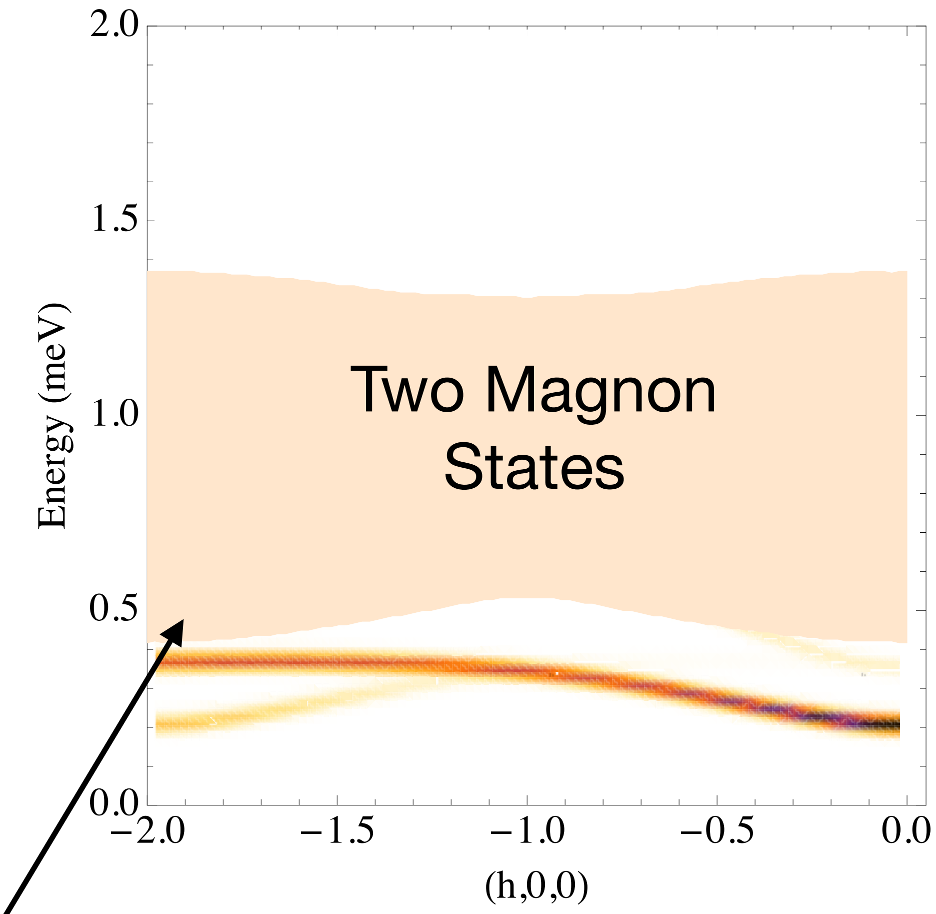
Four-magnon terms to same order.

Decay channels

Magnon damping kinematically constrained



$$\epsilon_{\boldsymbol{k}_3} = \epsilon_{\boldsymbol{k}_2} + \epsilon_{\boldsymbol{k}_1}$$

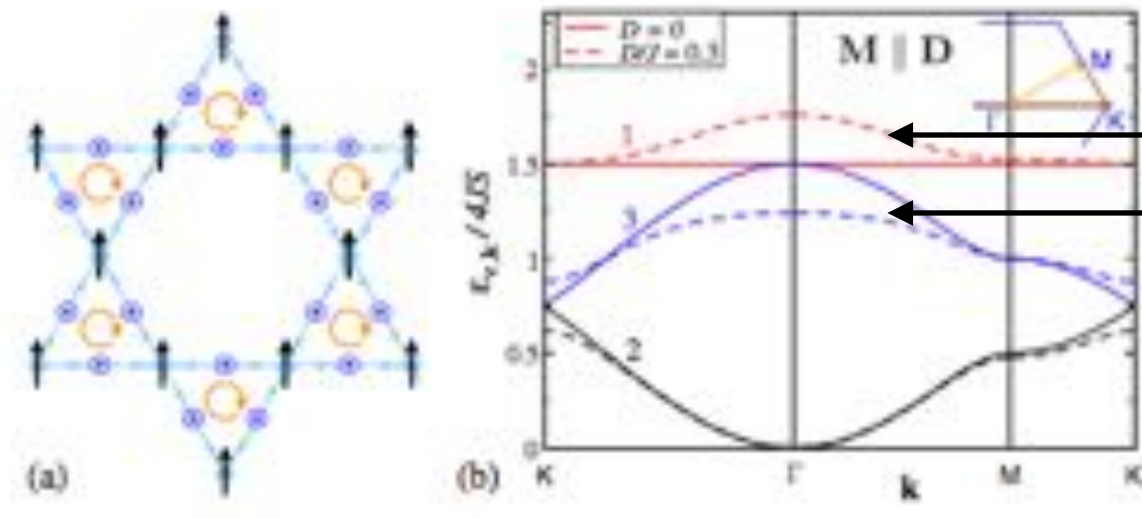


One-Two Magnon Overlap

Decay Kinematically Allowed

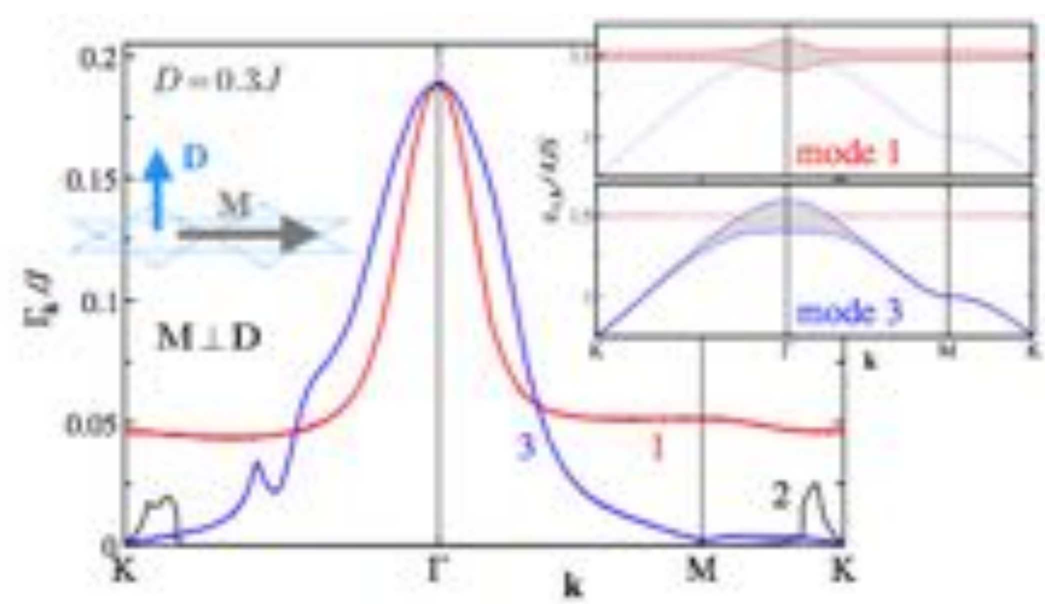
The Death of Topological Magnons?

Kagome ferromagnet with Dzyaloshinskii-Moriya exchange



Chernful bands

Large two magnon density of states in neighborhood of single magnon bands

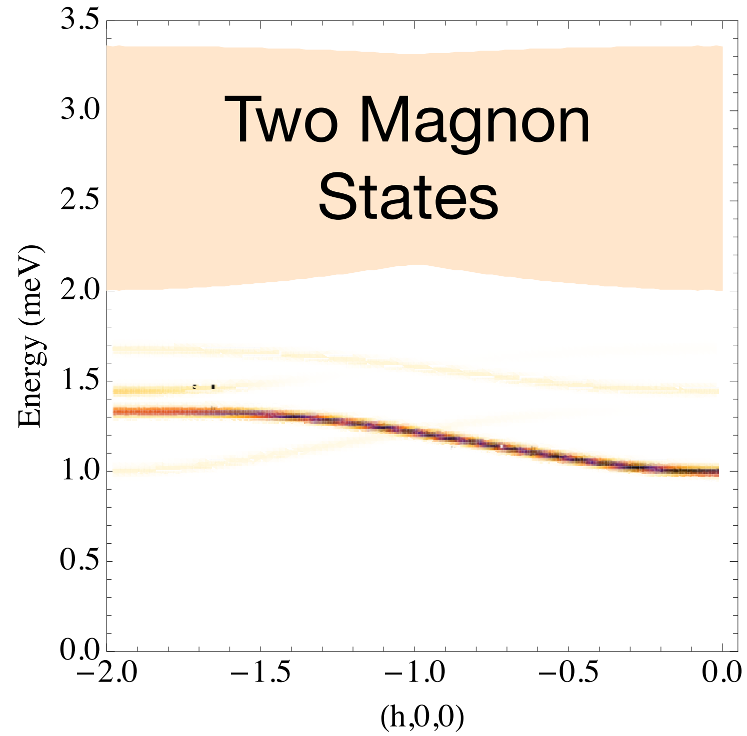


Mook, Menk, Mertig (2014)

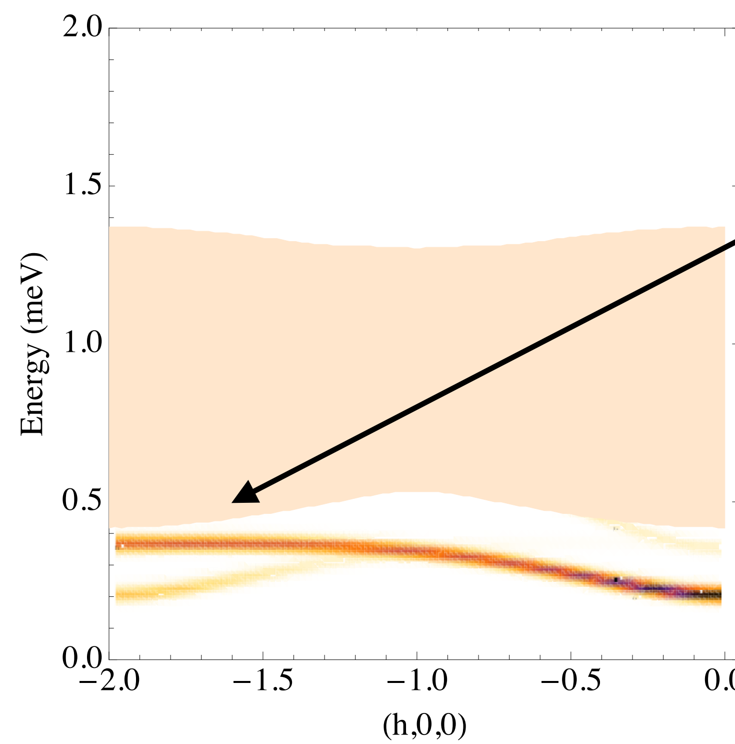
Chernyshev, Maksimov (2016)

Topological Magnons Live?

High Field



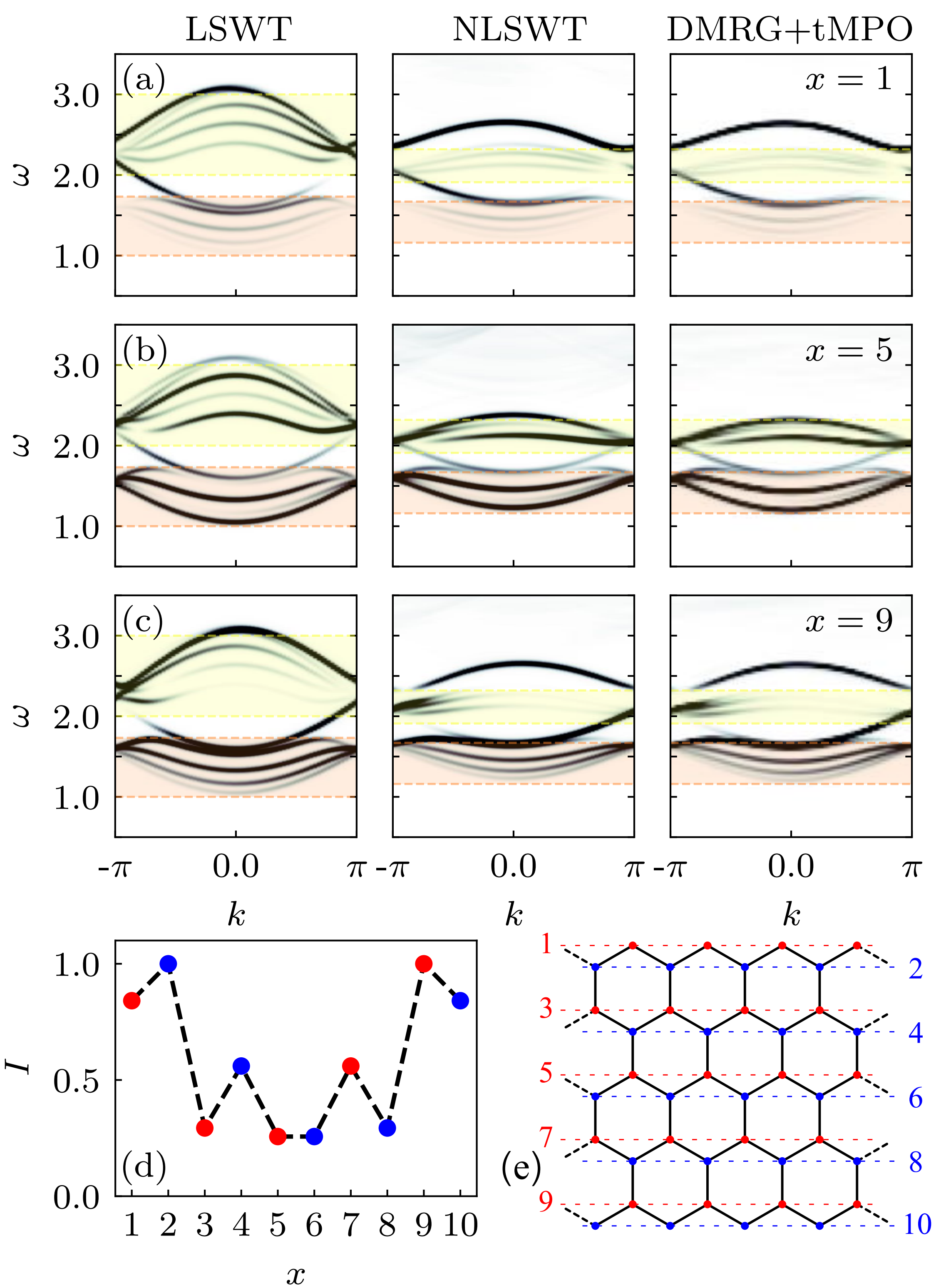
Low Field



One-Two Magnon Overlap

Decay Kinematically Allowed

Non-perturbative robustness of magnon chiral edge states



PHYSICAL REVIEW B 98, 060404(R) (2018)
Rapid Communications
Editors' Suggestion

Topological magnons in Kitaev magnets at high fields

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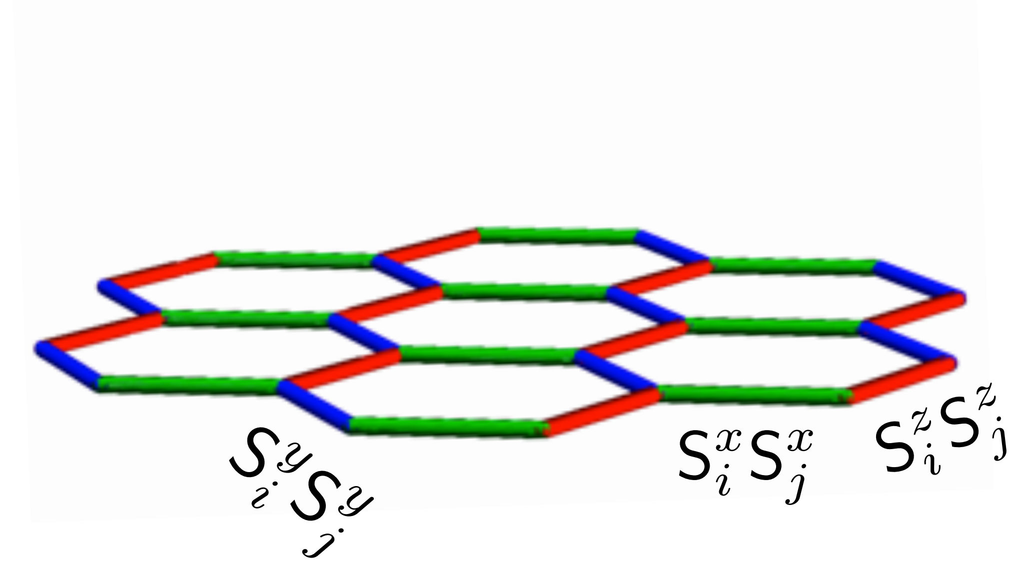
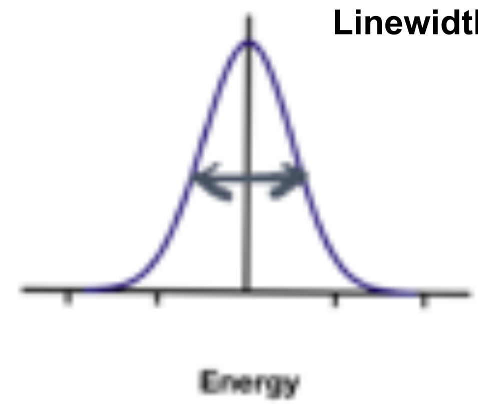
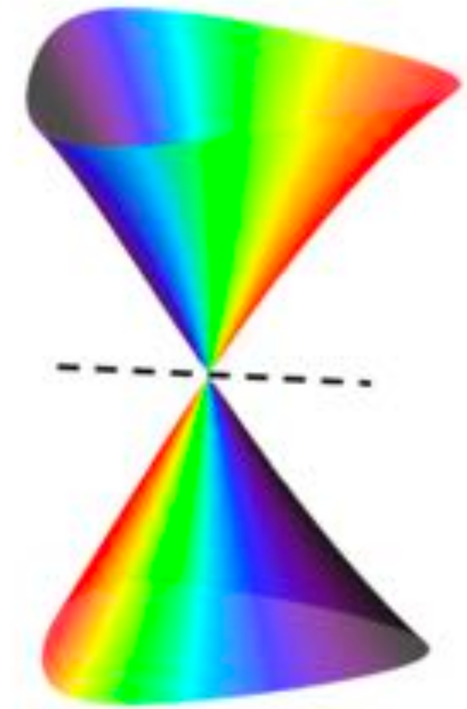
(Received 14 February 2018; published 7 August 2018)

Chern magnon bands appear generically in Kitaev honeycomb magnets in FM and field-polarized regimes



NonHermitian Topology in Magnons





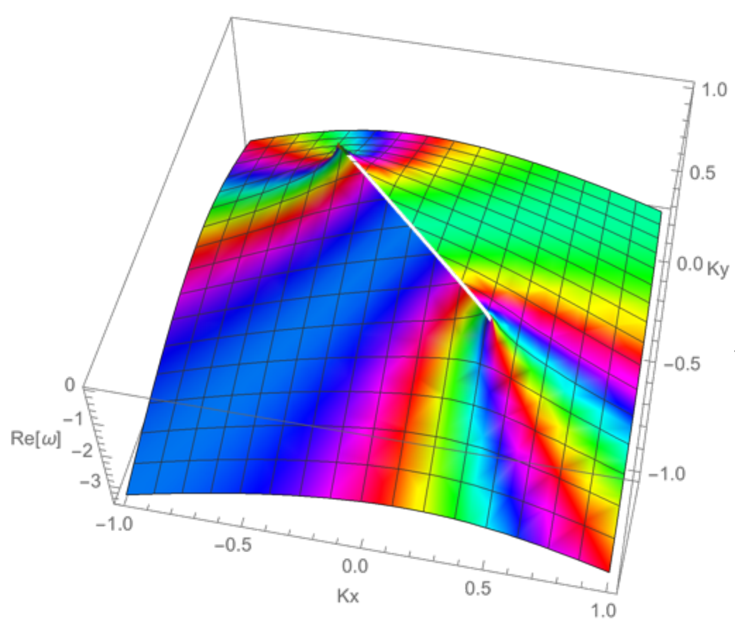
Lineshape winding and nonHermitian topology

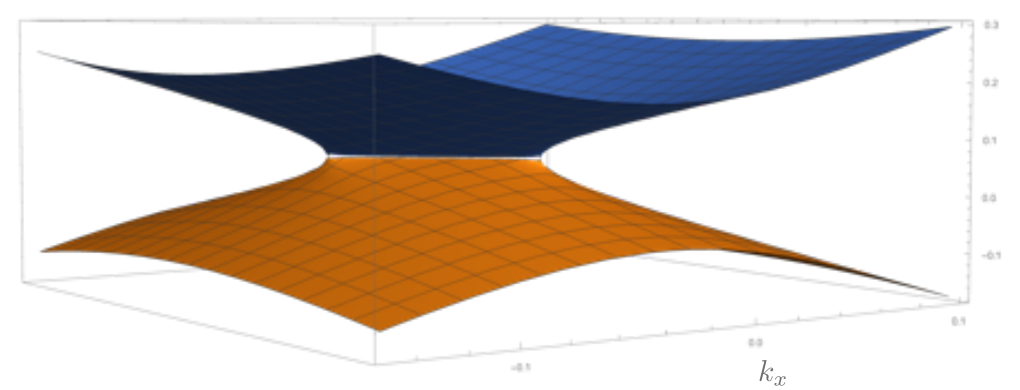
Number non-conservation from three-body terms

$$H_{\mathrm{NH}} = v(k_x\sigma_x + k_y\sigma_y + k_z\sigma_z) - i(a_0 + \boldsymbol{a}\cdot\boldsymbol{\sigma})$$

NonHermitian Hamiltonian coming from expansion around linear touching point with some self-energy

$$\boldsymbol{M}_{\mathrm{eff}}(\boldsymbol{k}) = \left[\boldsymbol{M}(\boldsymbol{k}) + \boldsymbol{\Sigma}'(\boldsymbol{k},\omega_0)\right] + \boldsymbol{\Sigma}''(\boldsymbol{k},\omega_0)$$

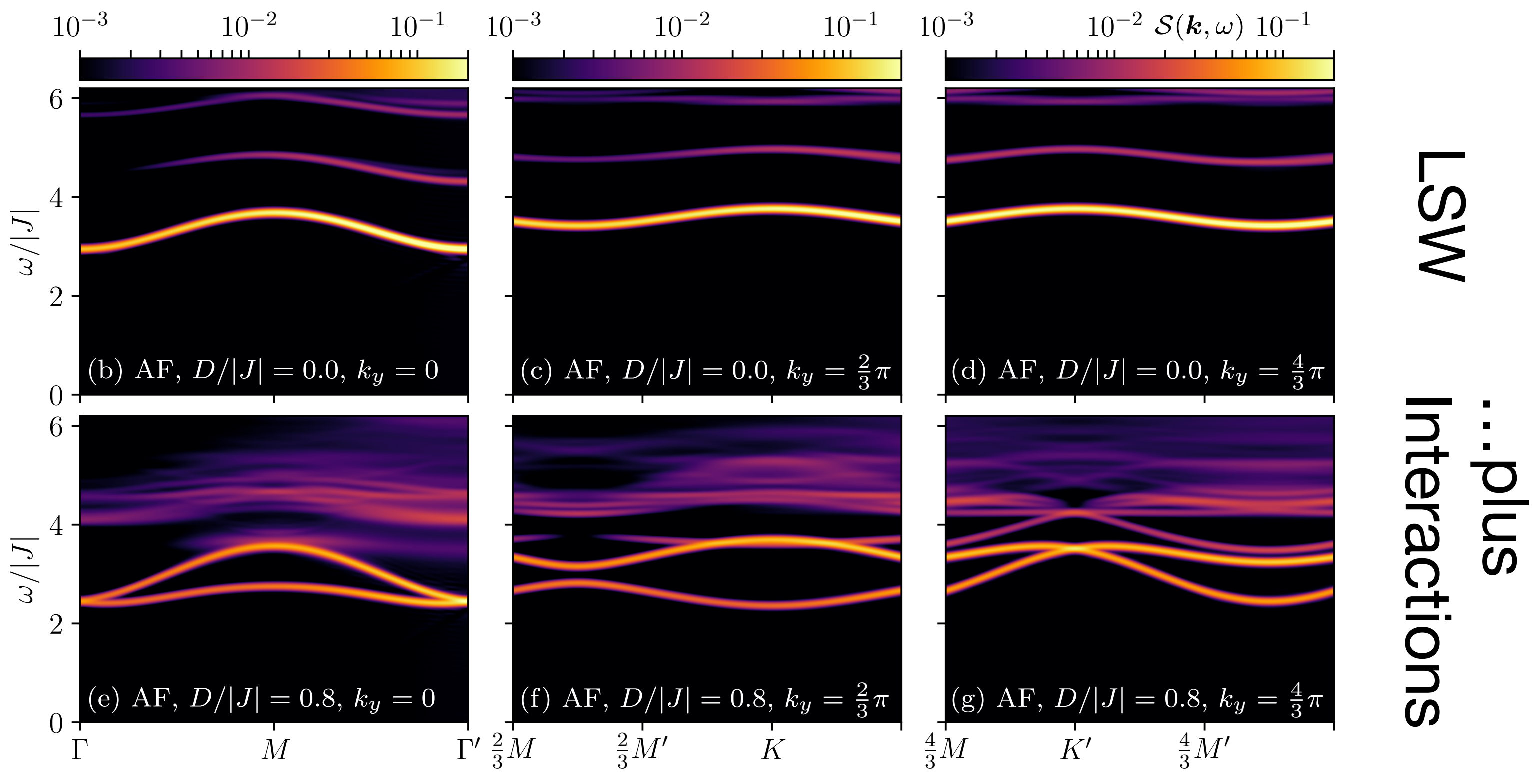




Winding of imaginary part of eigenvalue reflected in winding of spectral function - happens in anti-phase in upper and lower bands

PM, Rau, PRB (2019)

Interaction-induced Topology



Honeycomb lattice antiferromagnet with DMI:

$$H = \frac{1}{2}\sum_{\langle ij\rangle}\left[J_z S_i^z S_j^z + J\left(S_i^x S_j^x + S_i^y S_j^y\right) + \vec{D}_{ij}\cdot\vec{S}_i \times \vec{S}_j\right]$$

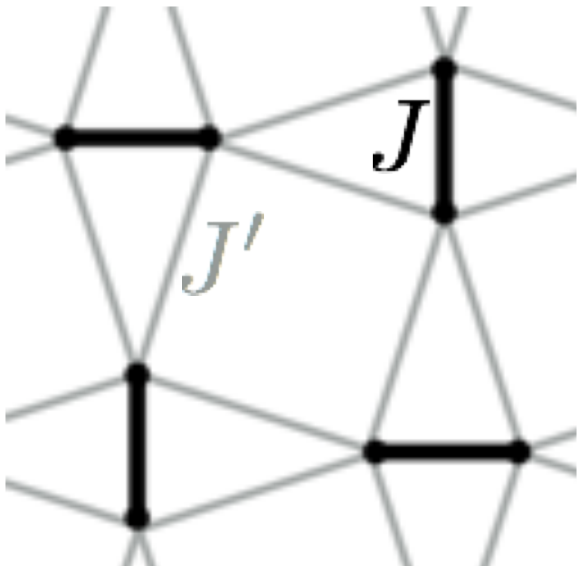
LSW insensitive to DMI which enters only as 3- and 4-body terms

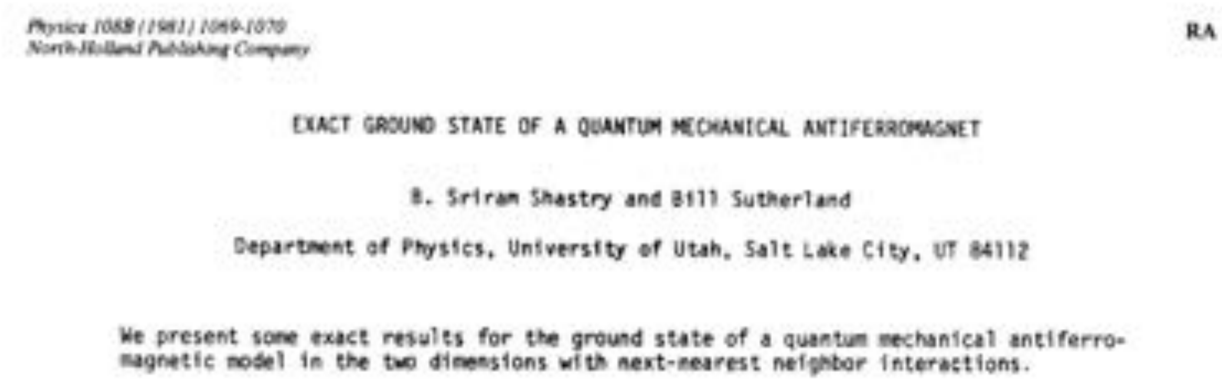
DMRG+tMPO reveals degeneracy breaking effects of DMI: leaving touching points at Γ and K'

Instance of **anisotropy blindness** of LSW theory resolved by including further neighbour couplings of equivalent symmetry

Gohlke, Corticelli, Moessner, PM, Mook, PRL 2023

Shastry-Sutherland: dimers and triplons





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North-Holland Publishing Company

RA 3

EXACT GROUND STATE OF A QUANTUM MECHANICAL ANTIFERROMAGNET

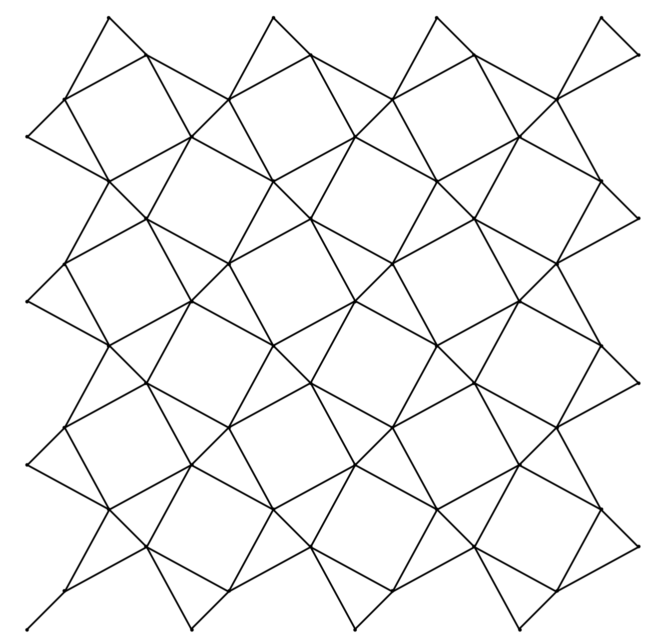
B. Sriram Shastry and Bill Sutherland

Department of Physics, University of Utah, Salt Lake City, UT 84112

We present some exact results for the ground state of a quantum mechanical antiferromagnetic model in the two dimensions with next-nearest neighbor interactions.

First/second neighbor isotropic antiferromagnetic exchange

$$\mathcal{H} = J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J' \sum_{\langle\langle i,j \rangle\rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$



Ground state is singlet tiling for $J'/J \lesssim 0.65$

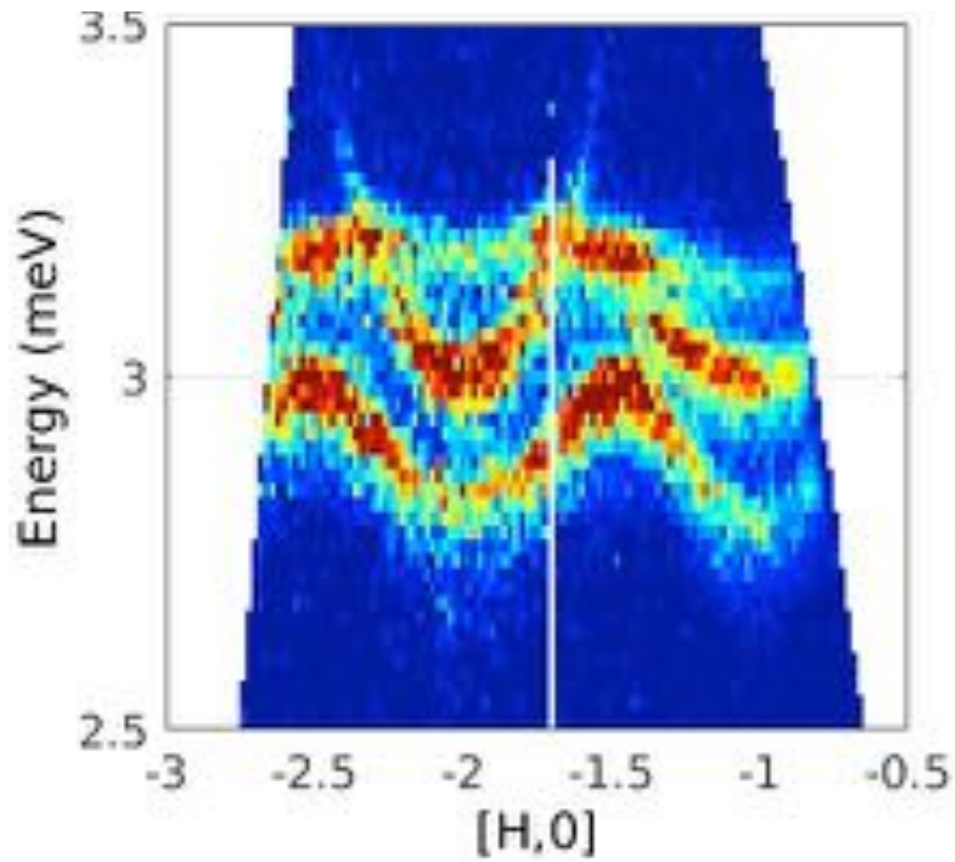
Triplon modes don't acquire dispersion up to $O((J'/J)^5)$

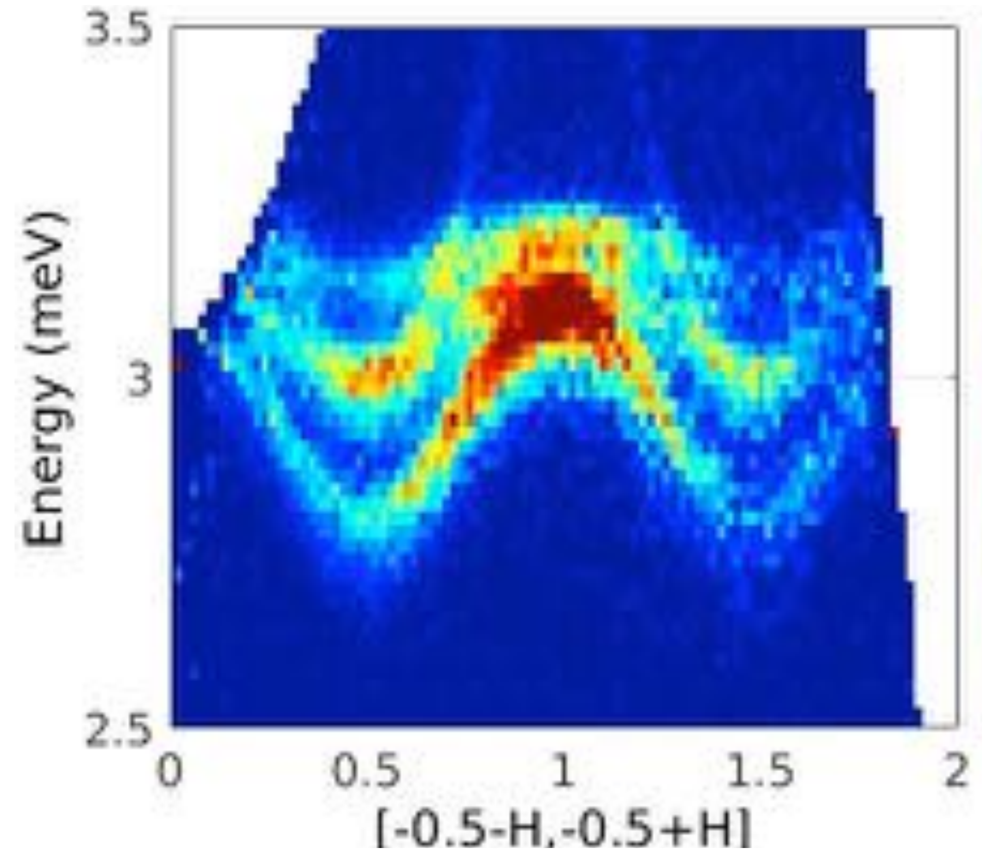
Three degenerate almost dispersionless triplon modes

Dispersion and degeneracy breaking mainly from Dzyaloshinskii-Moriya

Shastry-Sutherland and Topological Bound States

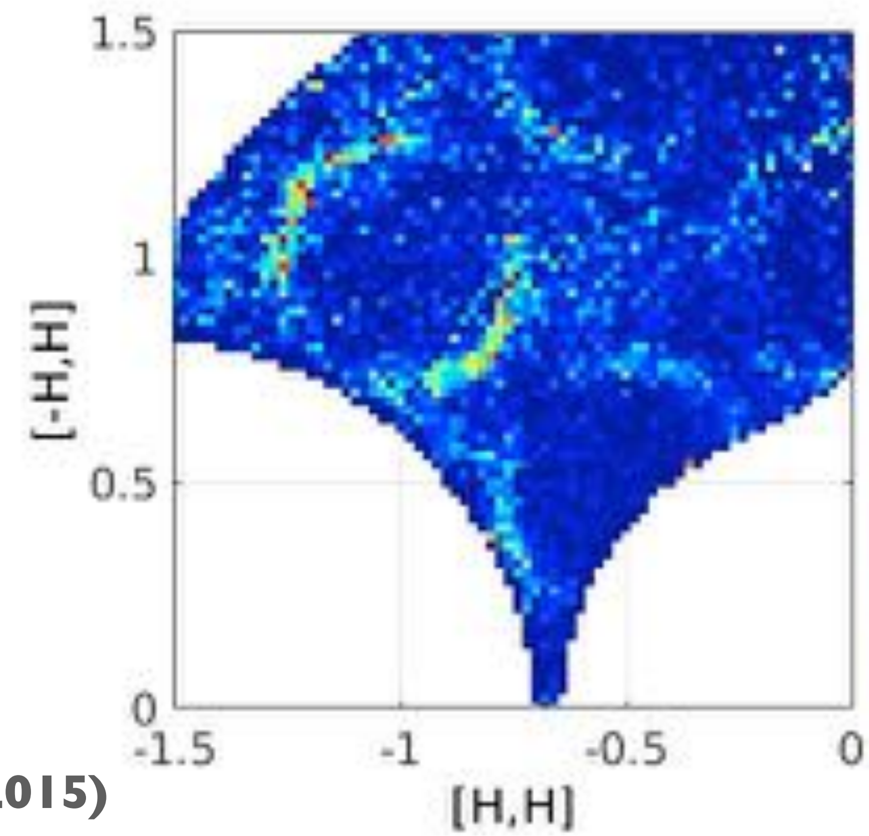
Optical mode with minimum around 3 meV. Parabolic dispersion at low energies.





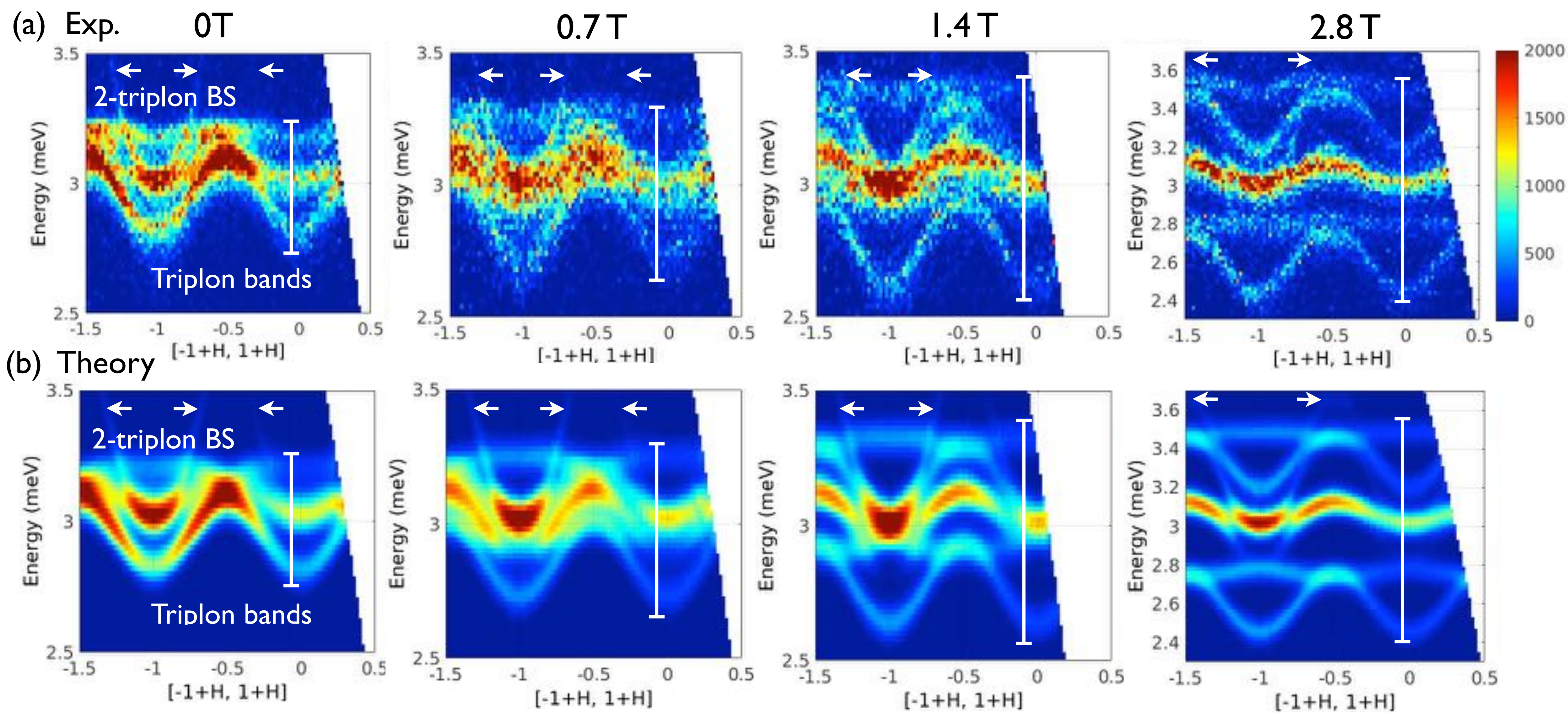
Meets Brillouin zone boundaries around 3.8 meV

Constant energy cut at 3.3 meV shows rings



Data taken on $SrCu_2(BO_3)_2$

Experiment inspired by Romhanyi, Penc, Ganesh, Nat. Comm. (2015)



Two triplon bound state hybridizes with single triplons - visible hybridization gaps

Result from theory: bound states inherit topology from single triplon modes

Sequence of topological transitions in magnetic field

PM et al. Nature Physics (2017)

Topological Magnons Outlook

Materials Discovery

- Progress in various directions but experiment lags theory
- Symmetry-based approaches make identification efficient
- Workflow: TQC to candidate materials to INS

  Large gap Chern, higher velocity Weyl points, multi-fold bosons, Chernful nodal planes, TCI

- Response of these systems

Surface States

- Various ideas: magnonic crystals and BLS as promising route to detection
- Gateway to exploring 2D magnets, possible manipulation and spintronics connections

Beyond topological magnons

- Spin-space groups in electronic systems e.g. altermagnets

Topology in strongly interacting systems

- Bound states can be topological
- Topology of multi-particle continua?
- Quantum spin liquids from topological magnon condensation?