Topological magnetic textures

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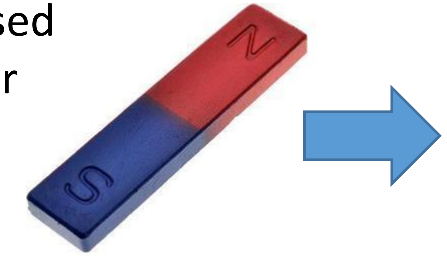
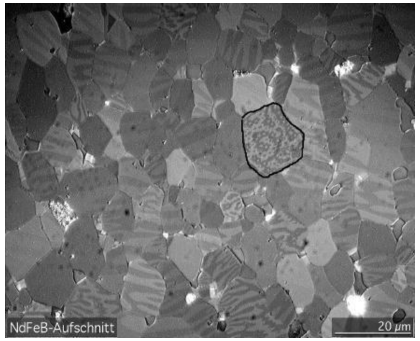
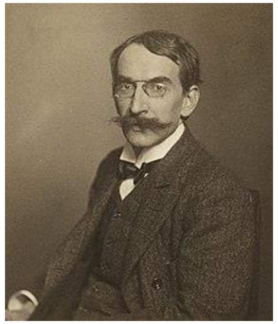
Introduction: magnetic textures

Ferromagnets can be composed of single magnetic domains or complex spin textures.

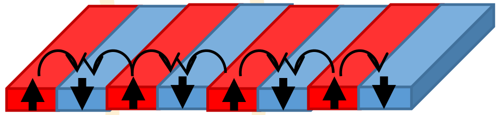


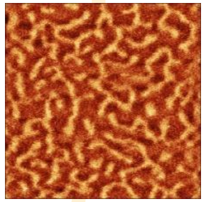


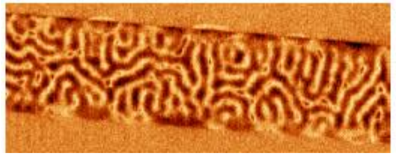


Pierre Weiss

Textured magnetic ground state
- Induced by crystal microstructure
- Induced by micromagnetic energy balance





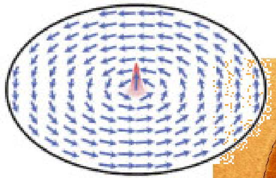


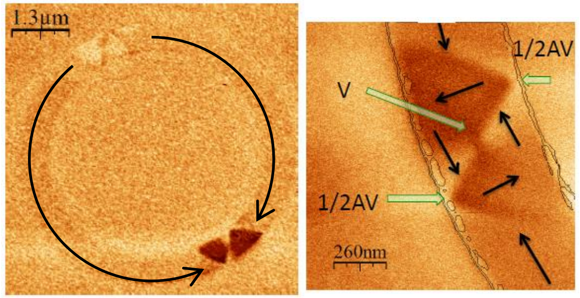
[Chauleau, SR PRB 2011]

Stripes stabilized by DMI + dipolar interaction in magnetic multilayer

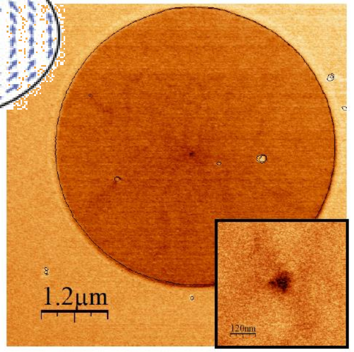
Complex spin texture in PdNi nanostructure stabilized by strain relief induced anisotropy and dipolar interactions

Metastable excitations => solitonic textures
- Vortex cores
- Magnetic domain walls
- Skyrmions
- …





Magnetic domain wall in NiFe stripe

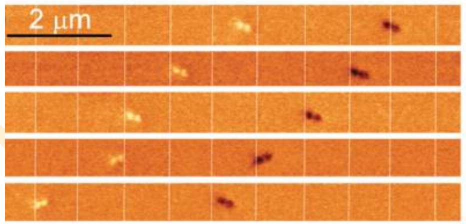


Magnetic vortex in NiFe disc

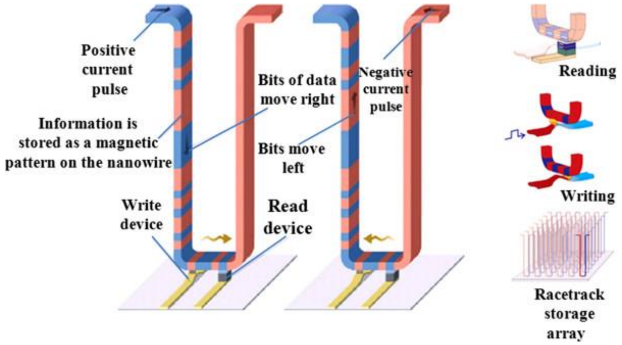
Size from few nanometers (vortex core, out-of-plance domain walls) to micrometers (in-plane domain walls)

Introduction: magnetic textures
Applications sin spintronics devices

Current induced domain wall motion
-> Race track memories (shift register) and logics

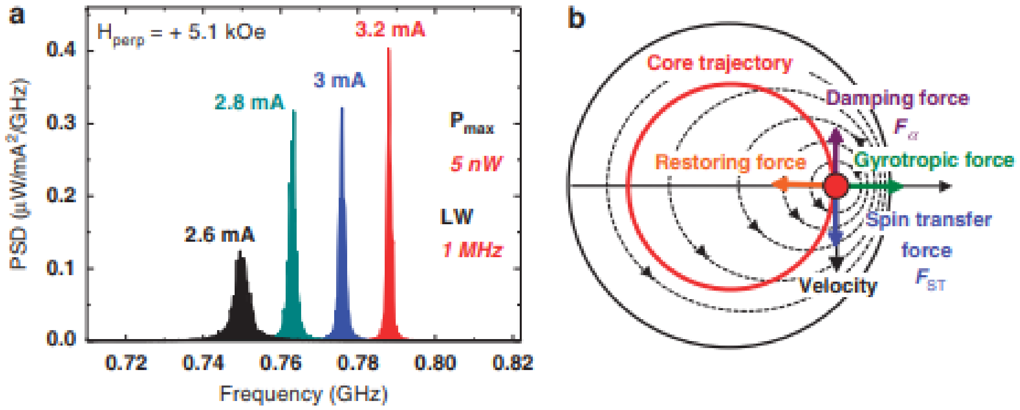


[Parkin et al. Science 2008]



Spin-polarized current through the magnetc material drives excitation (domain wall motion, vortex core precession)

Vortex core dynamics
-> Narrow linewidth oscillators (sub GHz)

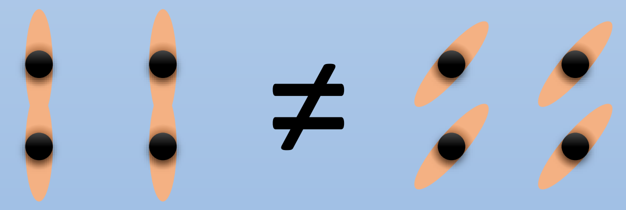


[Dussaux et al. Nature Com 2010]

… and future concepts using new textures for logics, neuromorphic and probabilistic computing, cryptography…

Introduction: micromagnetic framework

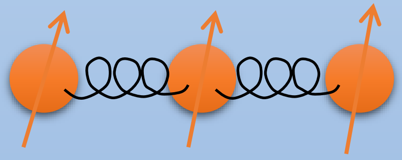
Magnetocrystalline anisotropy energy



$$E_{MC} = K(\vec{m}.\vec{e})^2$$

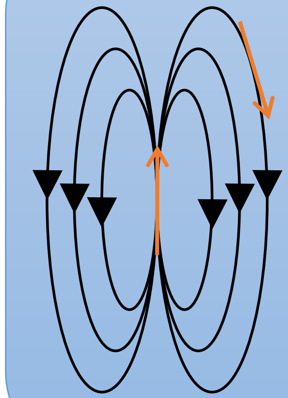
(simplest form, may be more complicated)
reflects the cristal symmetry

Exchange energy



$$E_{ex} = -J\vec{S}_i\vec{S}_j$$
$$= A(\nabla\theta)^2$$

Dipolar energy

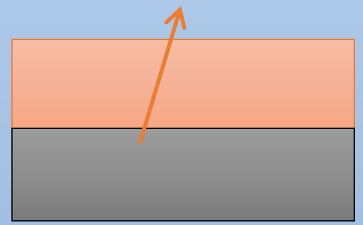


$$E_D = -\frac{\mu_0}{4\pi}\left(\frac{3(\vec{m}_i.\vec{r}_{ij})\vec{r}_{ij}}{r_{ij}^5} - \frac{\vec{m}_i}{r_{ij}^3}\right).\vec{m}_j$$

For practical use :
shape anisotropy

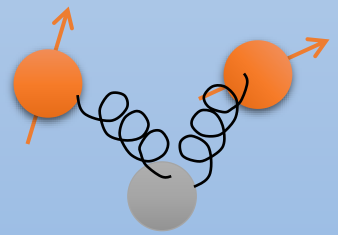
$$E_D = -\frac{1}{2}\mu_0\vec{M}.\vec{H}_d$$
$$= -\frac{1}{2}\mu_0\vec{M}.[N]\vec{M}$$

Interface anisotropy



$$E_{Interf.anis} = -\frac{K_S}{t}(\vec{m}.\vec{z})$$

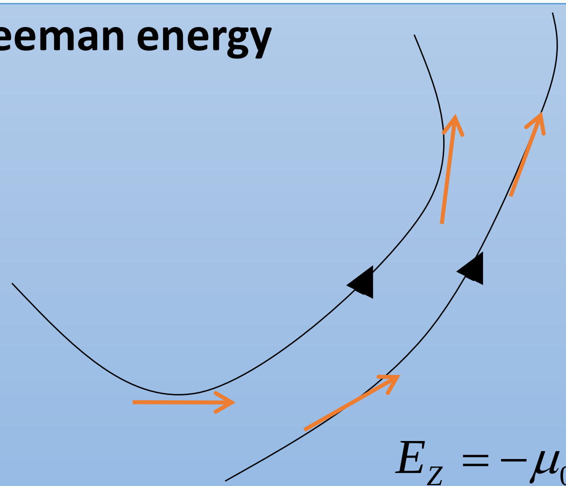
Dzyaloshinskii Moiya interaction



Requires absence of inversion symmetry (lattice or interface)

$$E_{DMI} = -\vec{d}_{ij}.(\vec{S}_i \times \vec{S}_j)$$

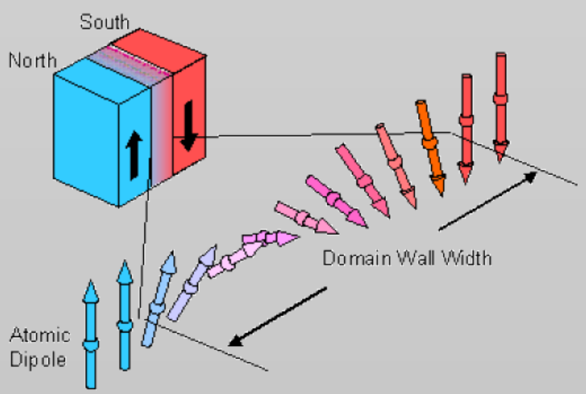
Zeeman energy



$$E_Z = -\mu_0\vec{M}.\vec{H}$$

Introduction: micromagnetic framework

Bloch wall
-> Anisotropy vs. Exchange



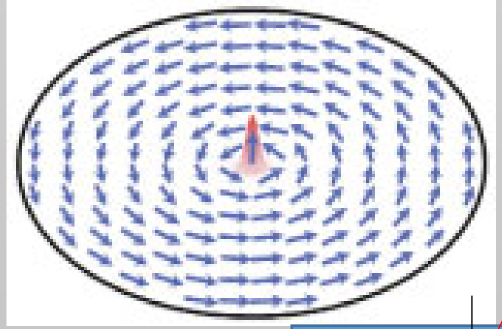
$$E = A\left(\frac{d\theta}{dx}\right)^2 + K\sin^2\theta$$

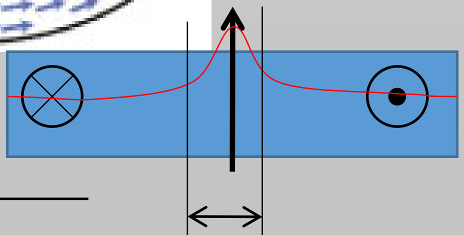
Length scale $\delta_B = \sqrt{A/K}$

Bloch wall energy $\sigma_B = 4\sqrt{AK}$

Exchange length
-> Dipolar coupling vs. Exchange

Ex : Magnetic vortex



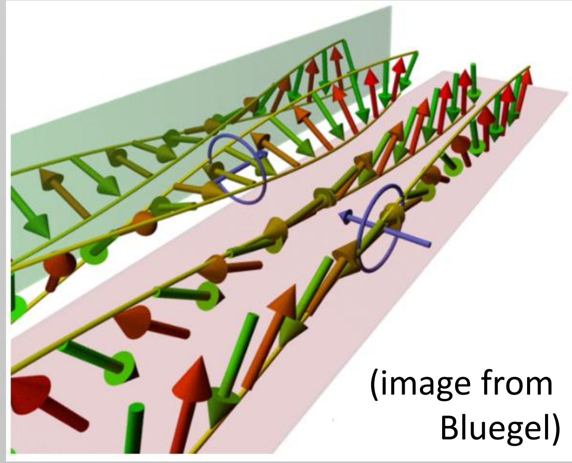


$$\Lambda = \sqrt{\frac{2A}{\mu_0 M_S^2}}$$

$\sim 2.6\Lambda$

Typical value : 5-10 nm

Dzyalinskii modulation
-> DMI vs. Exchange



(image from Bluegel)

Length scale $\xi = 2A/D$

Modulation period $2\pi\xi$

Quality factor

$$Q = \frac{2K}{\mu_0 M_S^2} = \left(\frac{\Lambda}{\delta}\right)^2$$

$Q > 1$ hard

$Q << 1$ soft

Why do we care about topology in magnetism ? Do we have properties that are directly related to topology ?

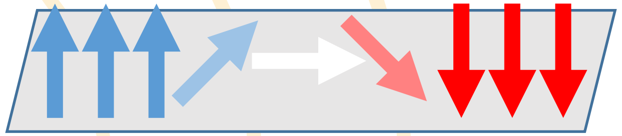
- Stability
- Dynamics

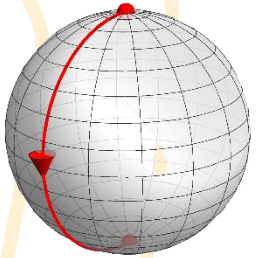
Texture stability

We consider a soliton-like texture (domain wall, skyrmion, vortex)

-> How can we transit towards a state with a different topology ?
-> How to relate energy with topology ?

Domain wall in a stripe



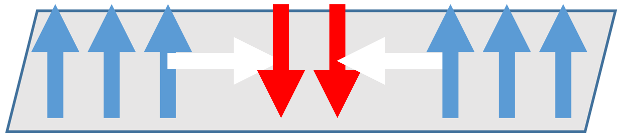


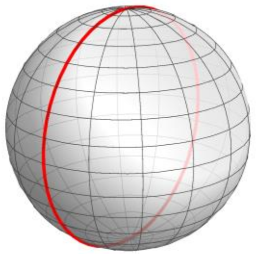
Topology is held by boundary condition

Stability requires to take the domain wall out of the stripe

=> Nothing interesting related to topology

360° domain wall

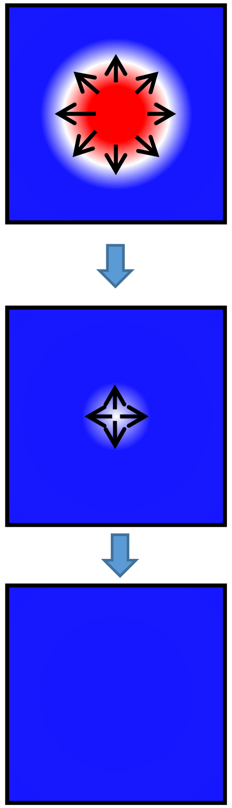




$\pi_1(\mathbb{S}^2)$ trivial topology

Stability is not related to topology but to specific energy terms (DMI, dipolar repulsion)

Skyrmion, magnetic vortex core

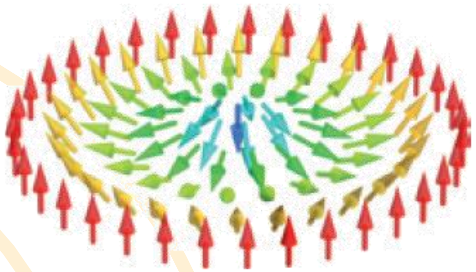


$\pi_2(\mathbb{S}^2)$ non-trivial topology

Skyrmion collapse or vortex core reversal goes through a topological defect ($\mathbb{S}^1$ vortex or $\mathbb{S}^2$ Bloch point)

Skyrmion

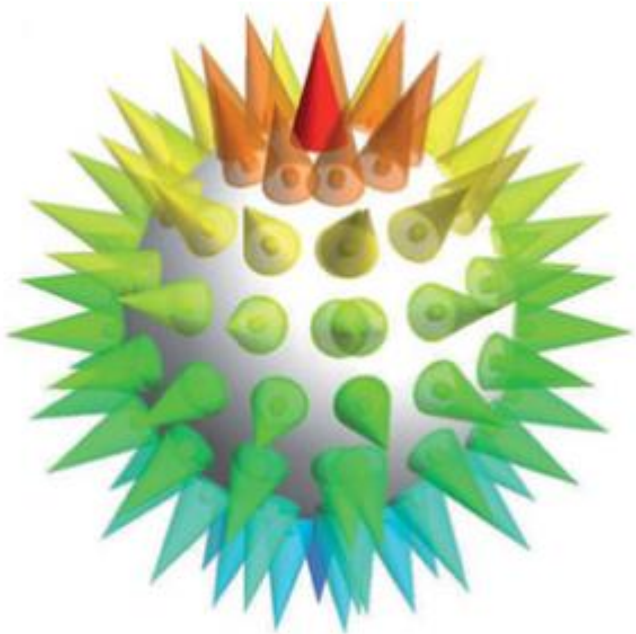
-> Chiral nanobubble



Néel Skyrmion (stabilized by interface DMI)

Bloch Skyrmion (stabilised by volume DMI in B20 crystals)

Order parameter space mapping:
Non trivial topology

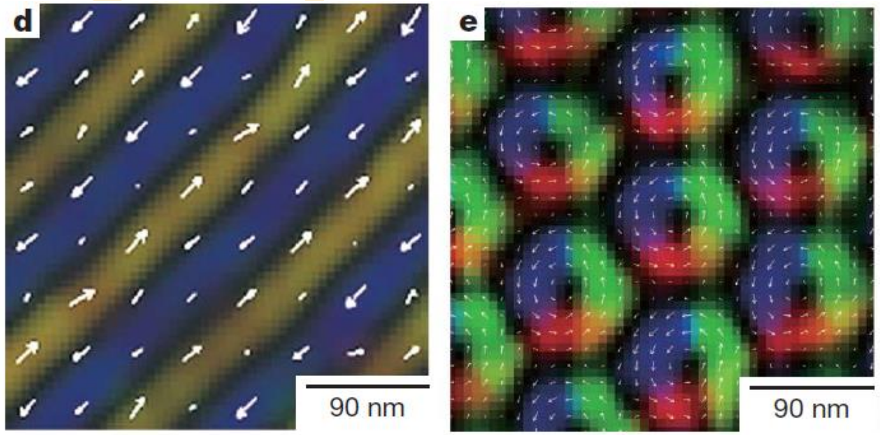


$$S = \frac{1}{4\pi} \int \vec{m} \left(\frac{\partial \vec{m}}{\partial x} \times \frac{\partial \vec{m}}{\partial y} \right) d^2 r = 1$$

$\Rightarrow$Impossible continuous transition toward the ferromagnetic phase (S = 0)

Skyrmions

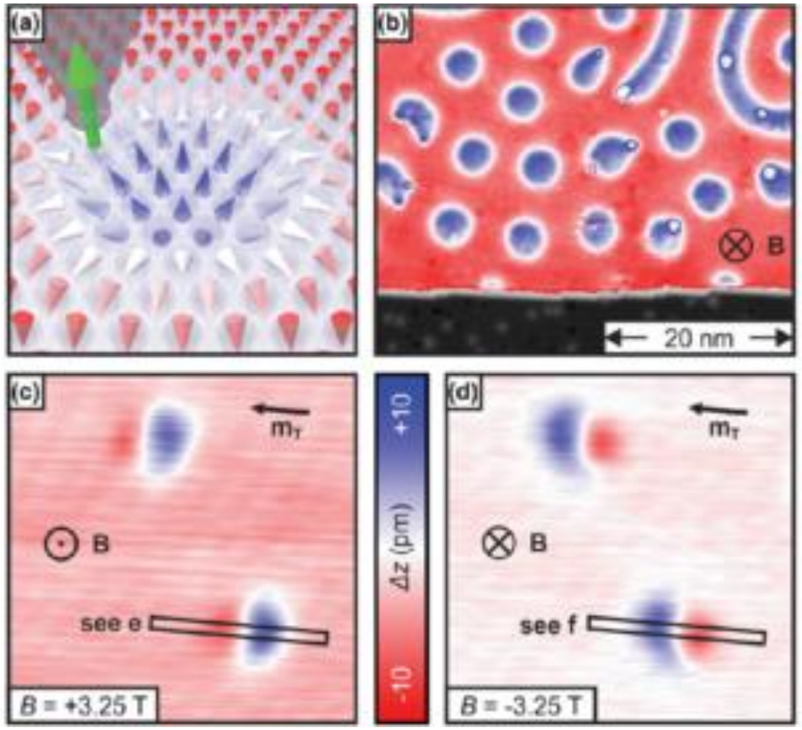
First images using TEM Lorentz imaging in $Fe_{0.5}Co_{0.5}Si$



X.Z. Yu et al. Nature 465, 901 (2010)

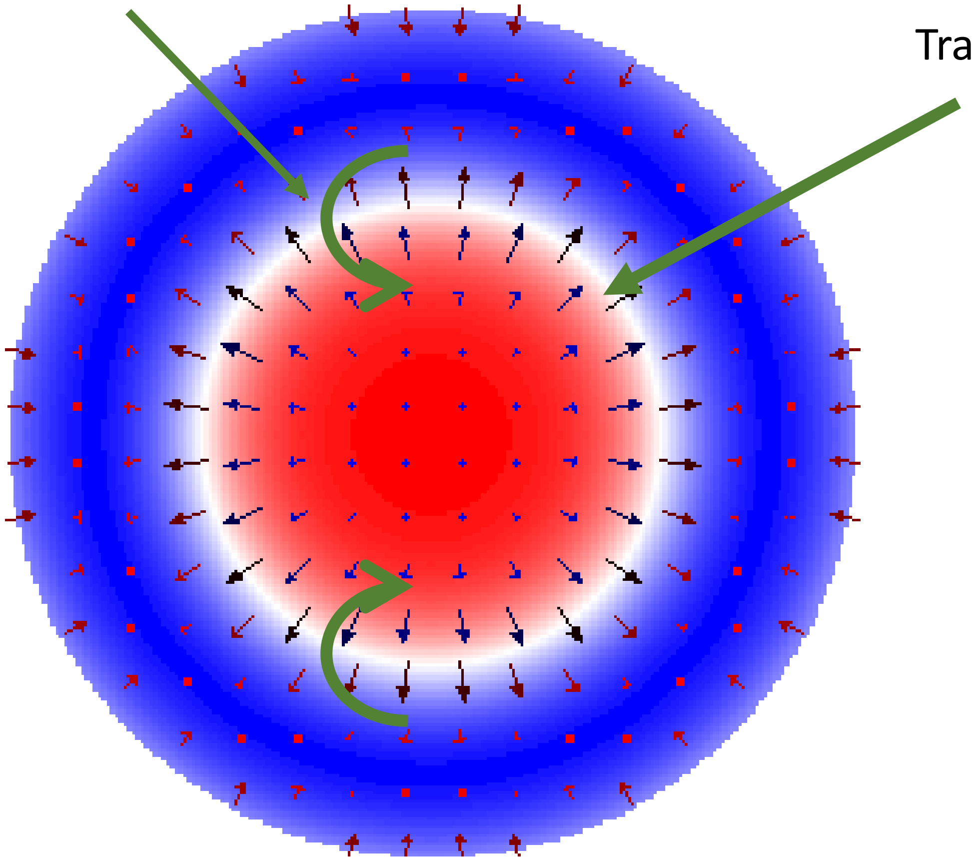
Interfaces stabilized skyrmions
Observed by SP-STM
Ir(111)/Fe(1ML)/Pd(1ML)

N. Romming et al. PRL 114, 177203 (2015)



Skyrmions: energy stabilization

Dipolar coupling: flux closure between core and surrounding



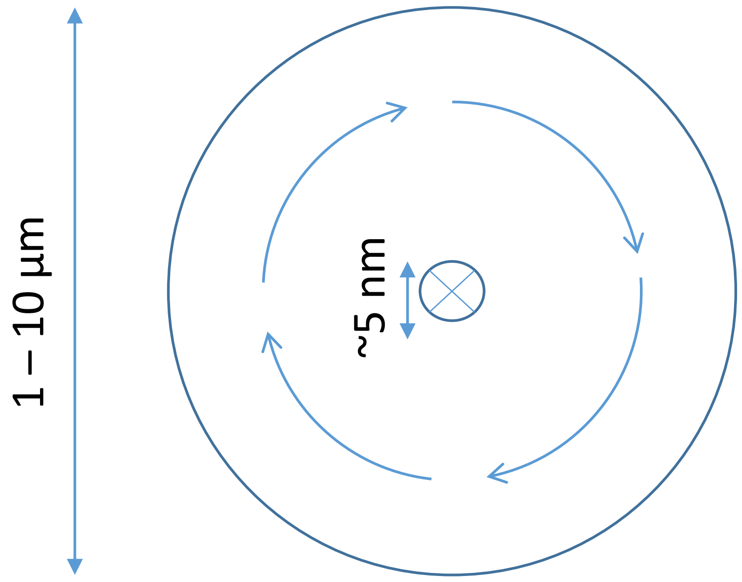
Transition : domain wall energy ($\sigma = 4\sqrt{AK} - \pi D$)
+ correction terms at smal size

Rohart and Thiaville
Phys. Rev. B (2013)

Magnetic Vortex

Soft magnetic disc (no anisotropy): need to minimize dipolar energy

- $\text{div}\,\vec{M} = 0$
- $\vec{M}.\vec{n} = 0$



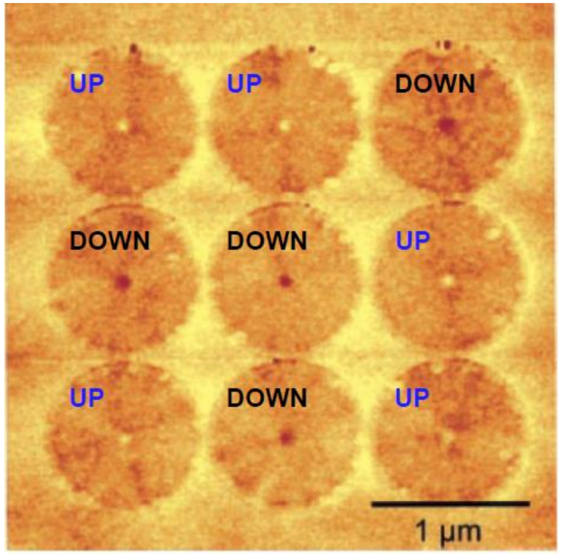


Fig. 2. MFM image of an array of permalloy dots 1 µm in diameter and 50 nm thick.

Shinjo et al. Science 289, 930 (2000)

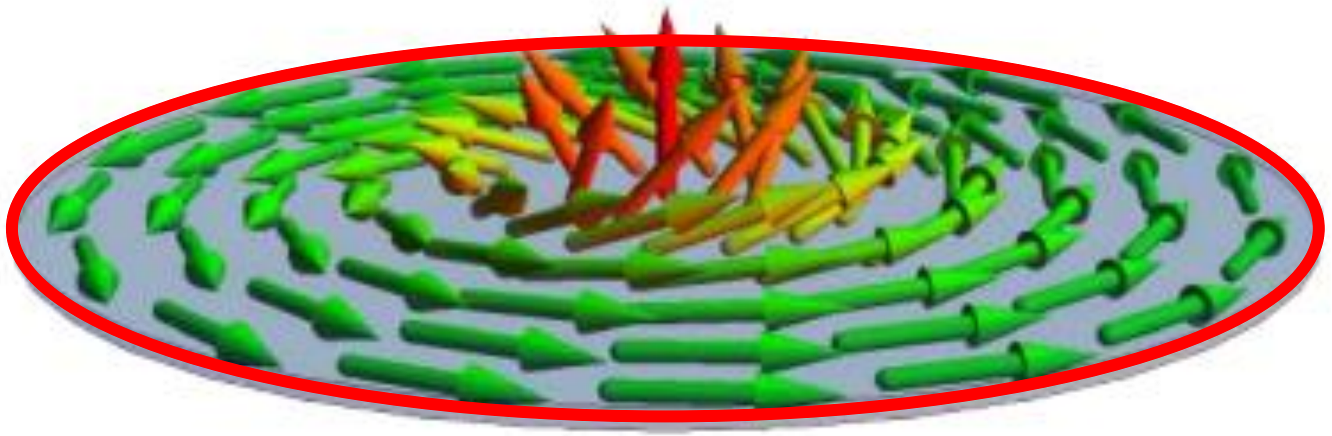
But exchange energy divergence at the center

$\Rightarrow$ Magnetization turns perpendicular over a distance $\Lambda = \sqrt{2A/\mu_0 M_S^2}$

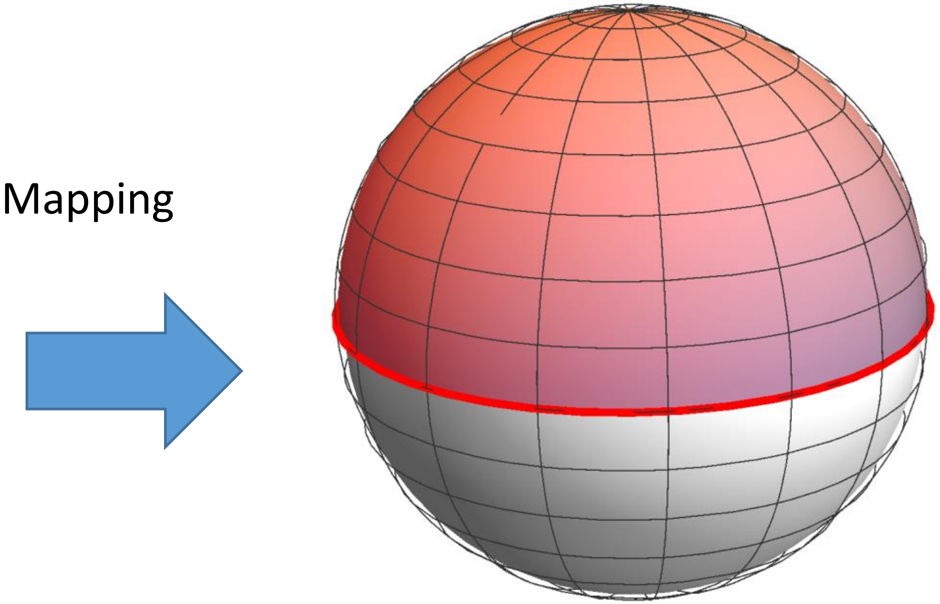
$\Rightarrow$ Vortex core

Beware: magnetic vortex leaves on $\mathbb{S}^2$.
It is not a XY (or $\mathbb{S}^1$) vortex (no topological defect at the center)
In topology, it can be refered to as a *meron*

Magnetic Vortex: topology



Mapping



Homotopy group : $\pi_2(\mathbb{S}^2)$

Sphere is covered once : $|n| = \frac{1}{2}$

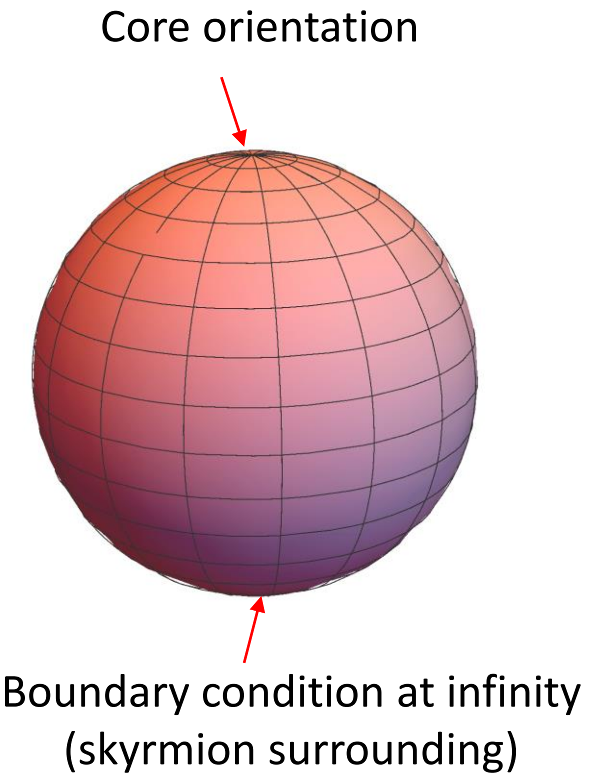
Topology depends on the vortex core orientation p and vorticity W (topology of te periphery)

$$n_{\pi_2(\mathbb{S}^2)} = pW = \pm\frac{1}{2}$$

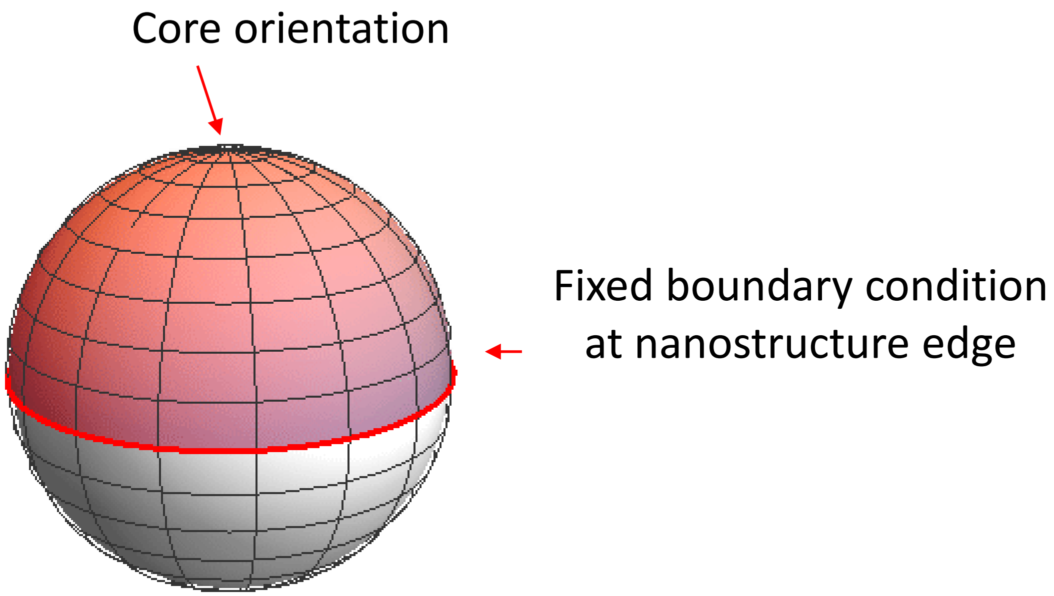
Stability of topological textures

- Colapse of skyrmions and vortex core switching require a change in topology

Skyrmion colapse



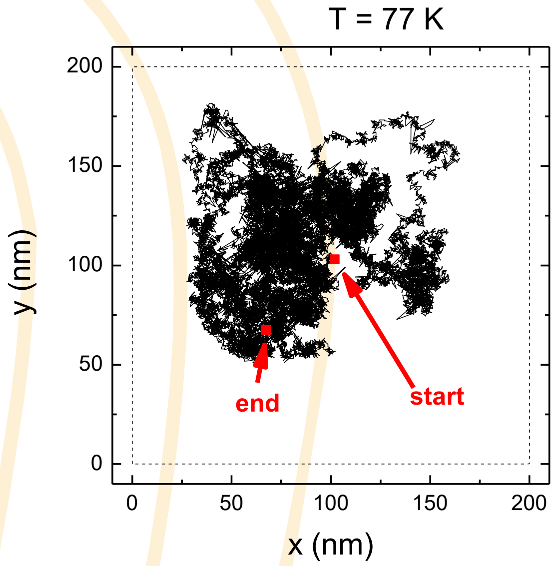
Vortex core switching

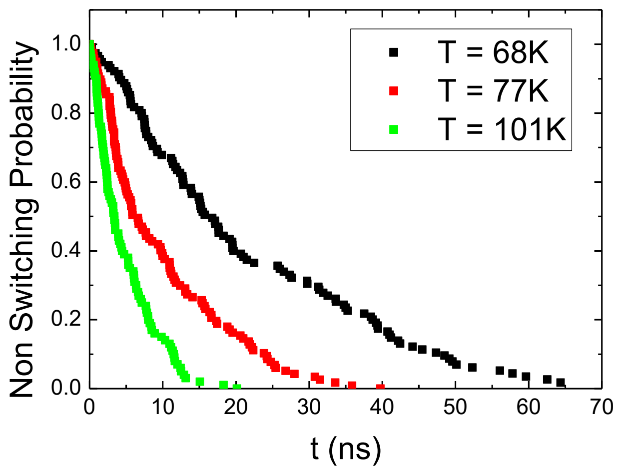


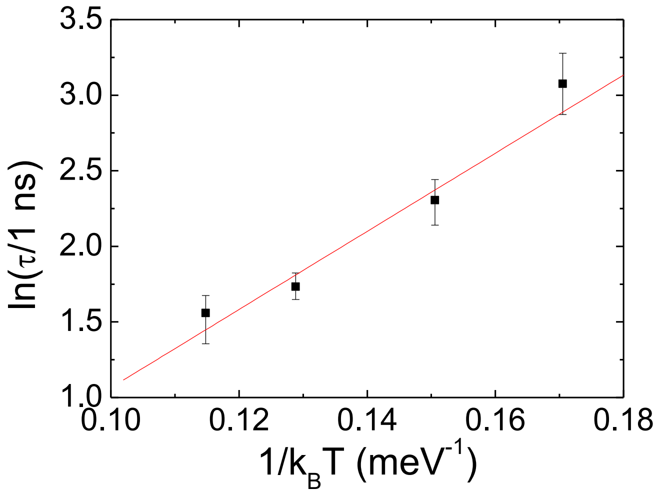
=> Requires the injection of a magnetic defect ($\mathbb{S}^1$ vortex or $\mathbb{S}^2$ Bloch point)

Stabilization of topological textures
skyrmion colapse

Finite colapse energy can be evidenced in simulations at finite temperatures







⇒ Example of a skyrmion in a Co monolayer on Pt(111)
Simulation at the atomic scale

Finite life time: Arrhenius low for the survival statistics
$t_{survival} = \exp -t/\tau$ and $\tau = \tau_0 \exp \Delta E/k_B T$
$\tau = 0.2$ ns, $\Delta = 27$ meV

Rohart et al. Phys. Rev. B 2016

Stabilization of topological textures skyrmion colapse

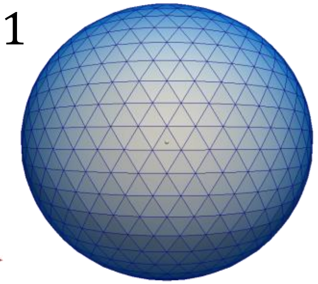
The topological problem doesn't exist at the atomic scale

Colapse path calculation

Nuged elastic band micromagnetics

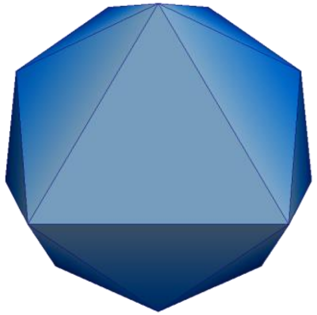
Initial state (skyrmion)

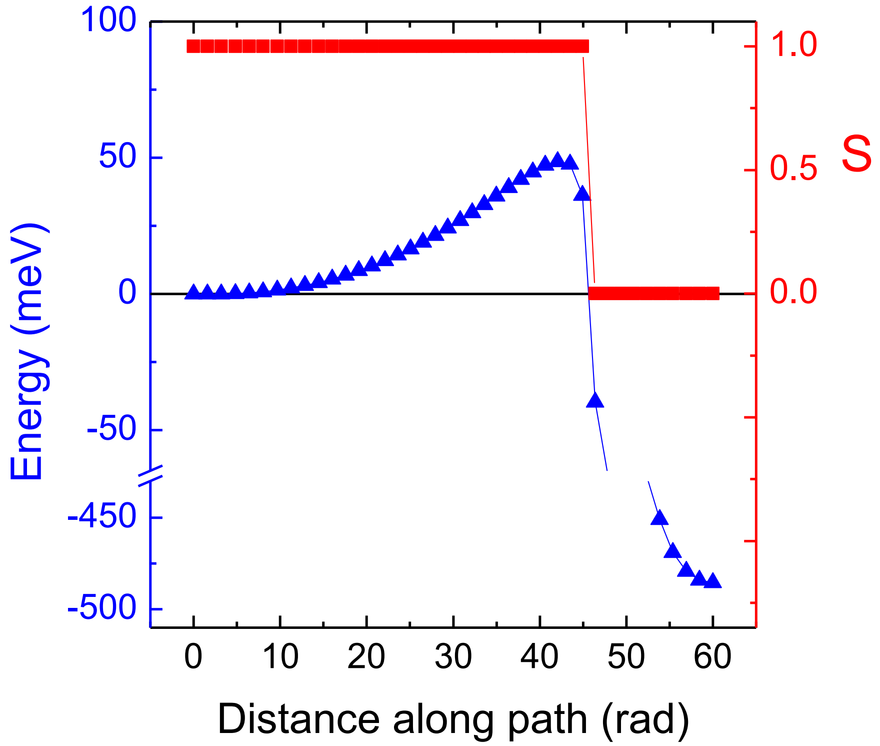
$n_{\pi_2(\mathbb{S}^2)} = 1$



Saddle point

$n_{\pi_2(\mathbb{S}^2)} \sim 0$



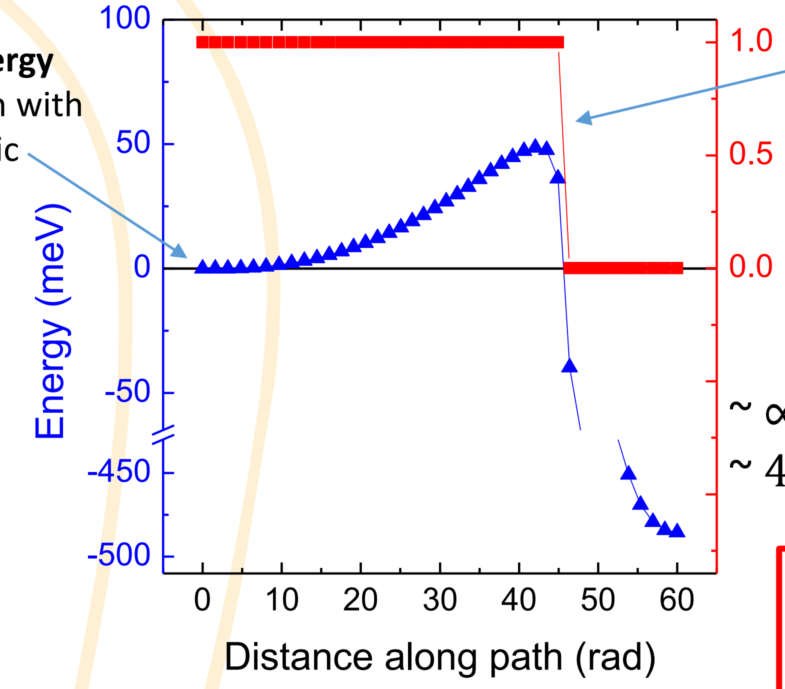


Rohart et al. Phys. Rev. B 2016

Stabilization of topological textures
skyrmion colapse

Can we understand the stability from micromagnetics arguments?

Skyrmion energy
=> Calculation with micromagnetic framework



Saddle point
⇒ Only a few atoms in the skyrmion core
Prevent from using the micromagnetic hypothesis

S Micromagnetic energy

$$E = 2\pi t \iint \left\{ A\left[\left(\frac{d\theta}{dr}\right)^2 + \frac{\sin^2\theta}{r^2}\right] - D\left[\frac{d\theta}{dr} + \frac{\cos\theta\sin\theta}{r}\right] + K\sin^2\theta - \mu_0 \overrightarrow{H_d}.\vec{M} \right\} r\,dr$$

~ ∝ skyrmion radius at large radius
~ $4\pi A t n_{\pi_2(\mathbb{S}^2)}$ at small sizes[1]

~ ∝ skyrmion radius

~ ∝ skyrmion core volume

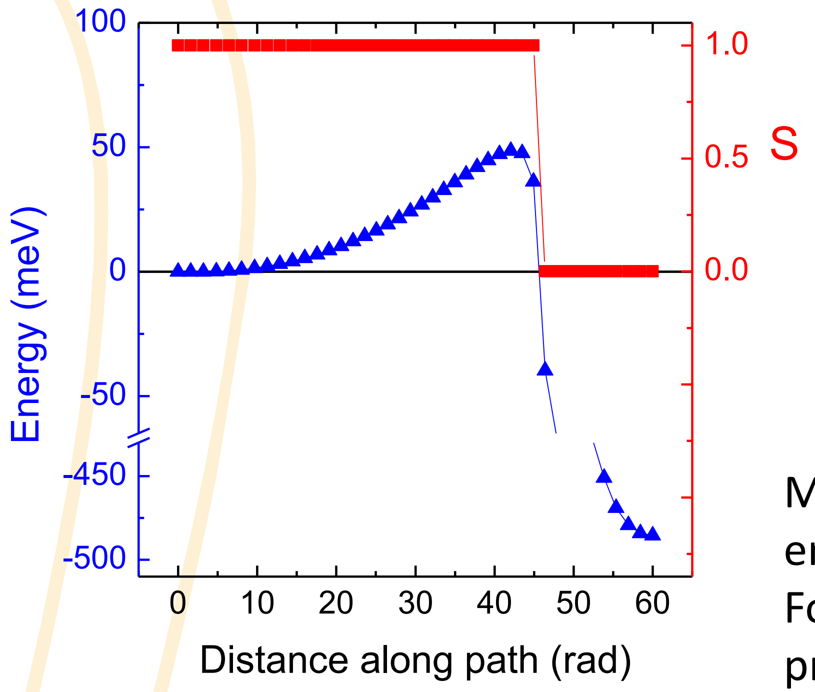
Colapse saddle point configuration: **energy is dominated by the exchange** whose energy cannot vanish due to topology

$$E_{saddle} \approx 4\pi A t |n_{\pi_2(S^2)}| = 4\pi A t$$

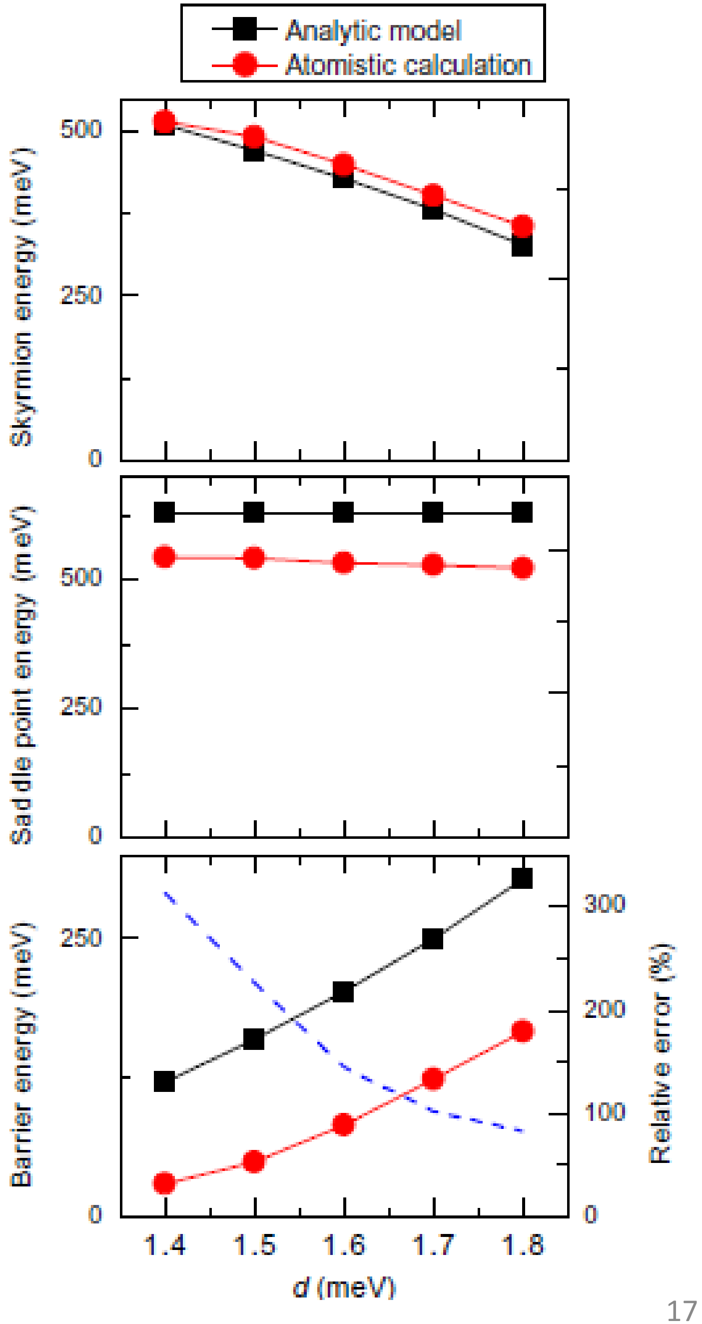
[1] [Belavin and Poliakov JETP Lett 22, 245 (1975)]
[Buttner et al. Sci. Rep. (2018)]

Stabilization of topological textures skyrmion colapse

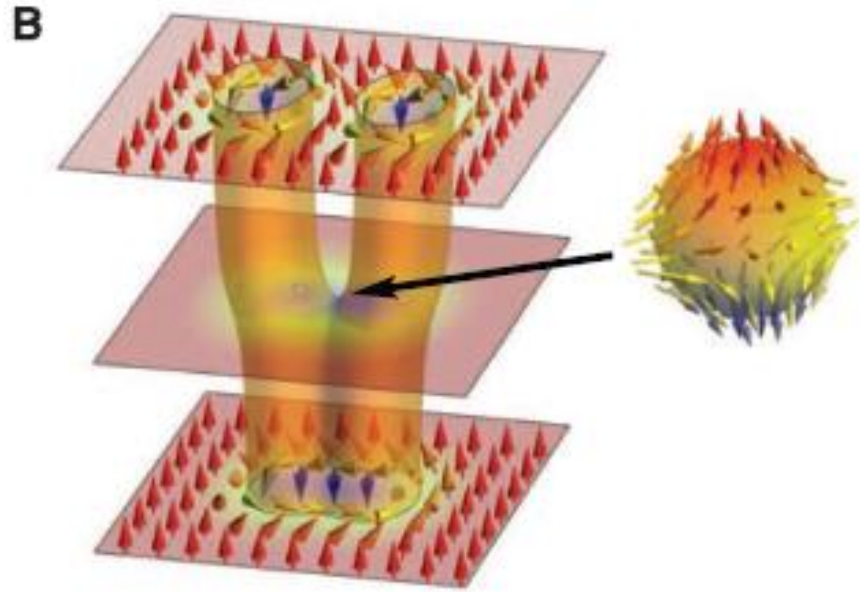
Can we understand the stability from micromagnetics arguments?

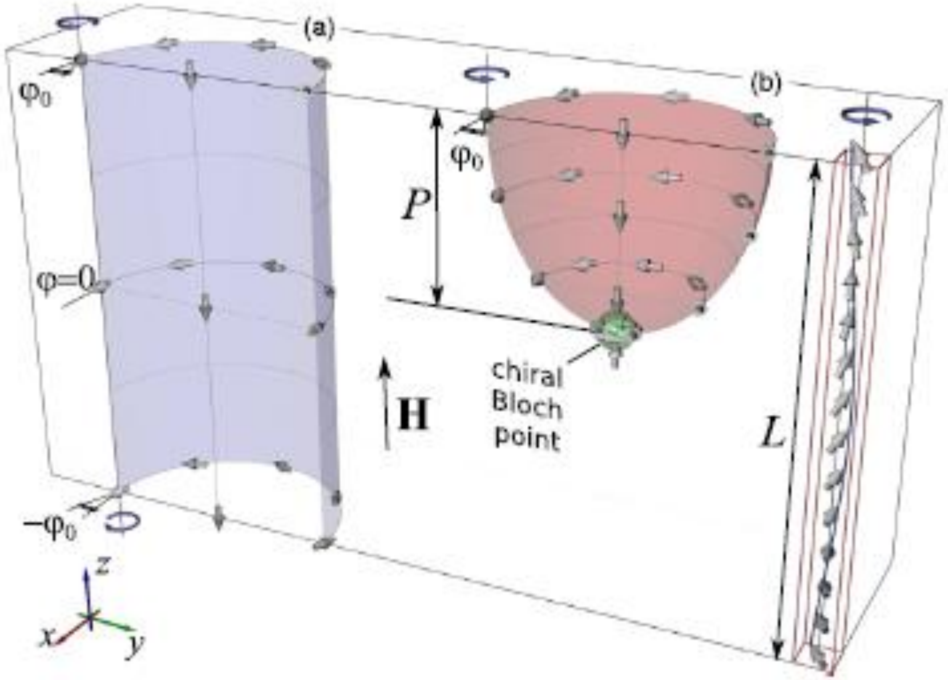


Micromagnetics can estimate the energies with a good accuracy.
For the energy barrier, the errors are propagating and the accuracy is poor



Stabilization of topological textures skyrmion colapse in thick samples





Collapse occurs via a Bloch (true 0D defect) point rather than a vortex (1D diffect in a thick sample)

From *Topological defect-mediated skyrmion annihilation in three dimensions*
Birch et al. Comm. Phys. 4, 175 (2021)

See also Milde et al. *Unwinding of a skyrmion lattice by magnetic monopoles*
Science 340, 1076 (2013)

Stabilization of topological textures vortex core reversal

Switching the vortex core modifies the topology from $S = \frac{1}{2}$ to $S = -\frac{1}{2}$.

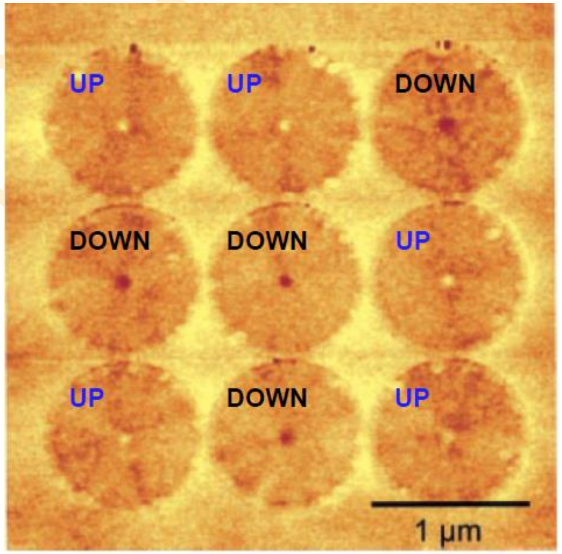


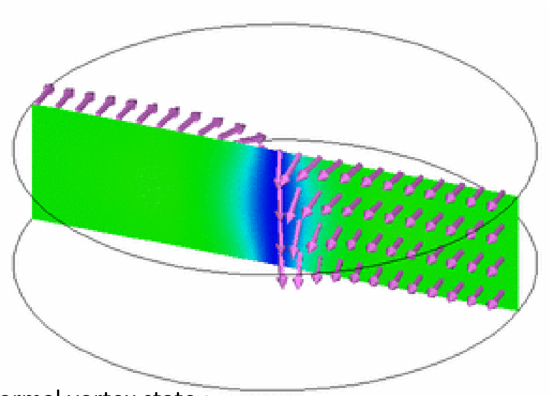
Fig. 2. MFM image of an array of permalloy dots 1 µm in diameter and 50 nm thick.

Shinjo et al. Science 289, 930 (2000)

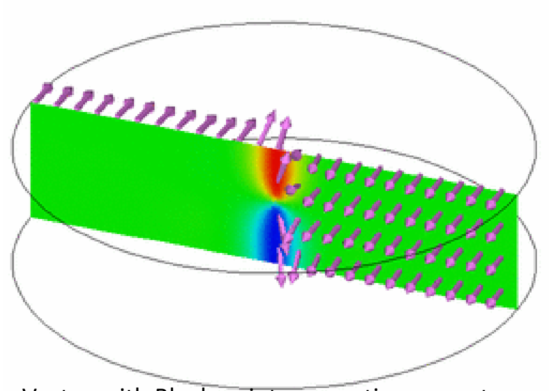
Thick problem ($t > \Lambda = \sqrt{2A/\mu_0 M_S^2}$):

magnetization is not constant along the z direction.

Vortex core switching is not homogeneous: nucleation of a Bloch point.



Normal vortex state : magnetization is perpendicular to minimize exchange energy

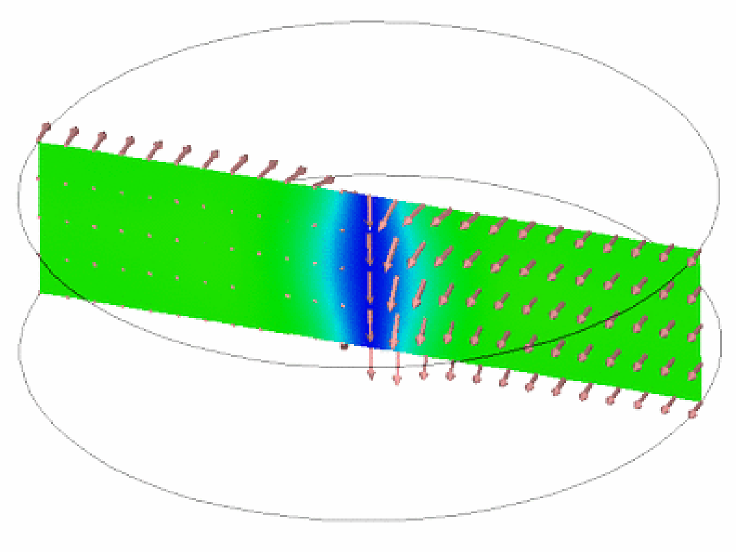


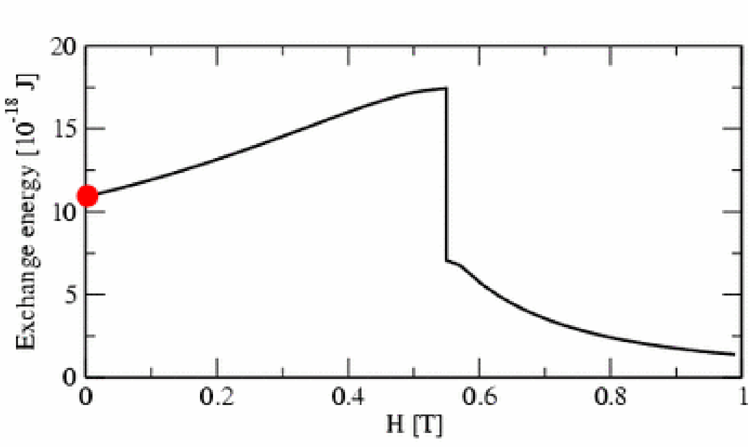
Vortex with Bloch point: magnetic moment are all almost in plane**, mean magnetization is zero at the core**

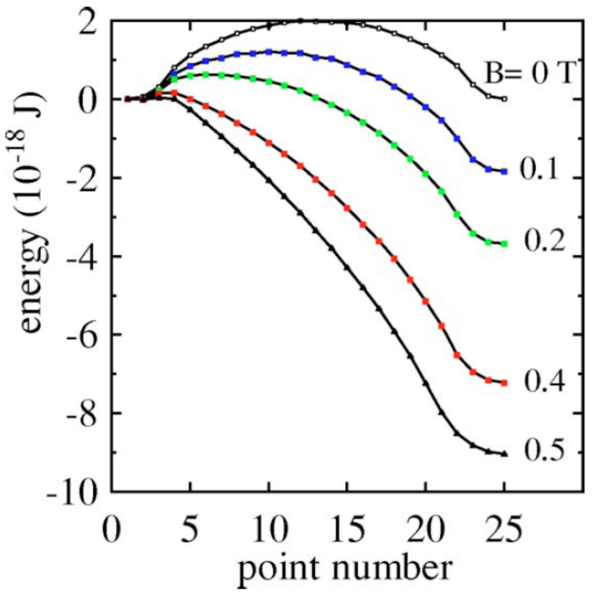
Thiaville et al. PRB 67, 094410 (2003)
Images from R. Dittrich http://magnet.atp.tuwien.ac.at/gallery/bloch_point/index.html

Stabilization of topological textures
vortex core reversal

Switching the vortex core modifies the topology from $S = \frac{1}{2}$ to $S = -\frac{1}{2}$.







Thiaville et al. PRB 67, 094410 (2003)
Images from R. Dittrich http://magnet.atp.tuwien.ac.at/gallery/bloch_point/index.html

Dynamics: basis of magnetization dynamics

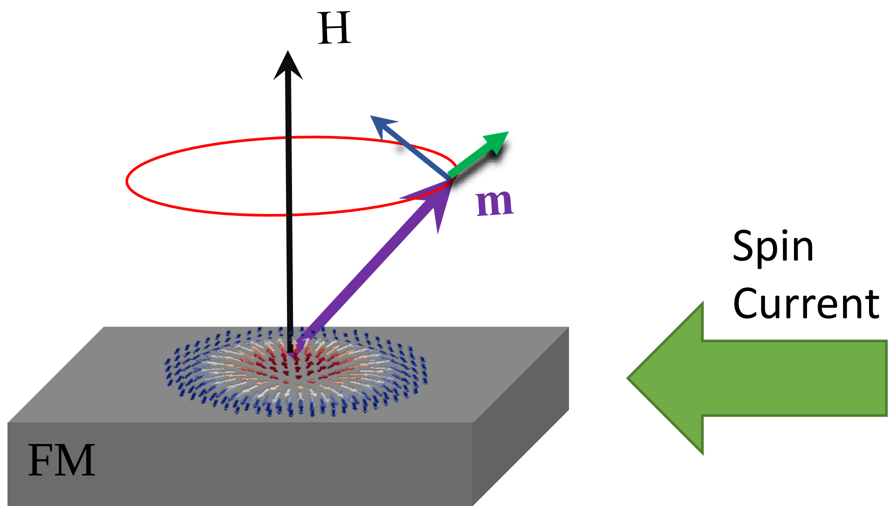
Ferromagnetic dynamics

Landau-Lifshitz-Gilbert (LLG) equation:

$$\frac{\partial \boldsymbol{m}}{\partial t} =$$

Precession **Damping** **Spin transfert torque**

L. Landau and E. Lifshitz. Phys. Z. sowjetunion 8 153 (1935).



Intergration over space: texture dynamics

Thiele equation

$$\boldsymbol{G} \times \boldsymbol{v} - \alpha D \boldsymbol{v} + \boldsymbol{F}_{\boldsymbol{STT}} = 0$$

Thiele equation

Integrated over the whole space assuming no deformation

Ref. STT
I. M. Miron *et al. Nature*, 476 (2011).

A.A. Thiele. *Phys. Rev. Lett.*, 30, (1973).

Dynamics of topological textures
Thiele equation

$$\vec{G} \times \vec{v} - \alpha \bar{\bar{D}} \vec{v} + \vec{F}_{ext} + \vec{F}_{STT} = \vec{0}$$

Gyrotropic deflection Dissipation External potential Current induced force

$$\vec{G} = -\frac{\mu_0 M_S t}{\gamma_0} 4\pi \, n_{\pi_2(\mathbb{S}^2)} \, \vec{z}$$

$$D_{ij} = \frac{\mu_0 M_S t}{\gamma_0} \iint \left(\frac{\partial \vec{m}_0}{\partial i} . \frac{\partial \vec{m}_0}{\partial j} \right) d^2 r$$

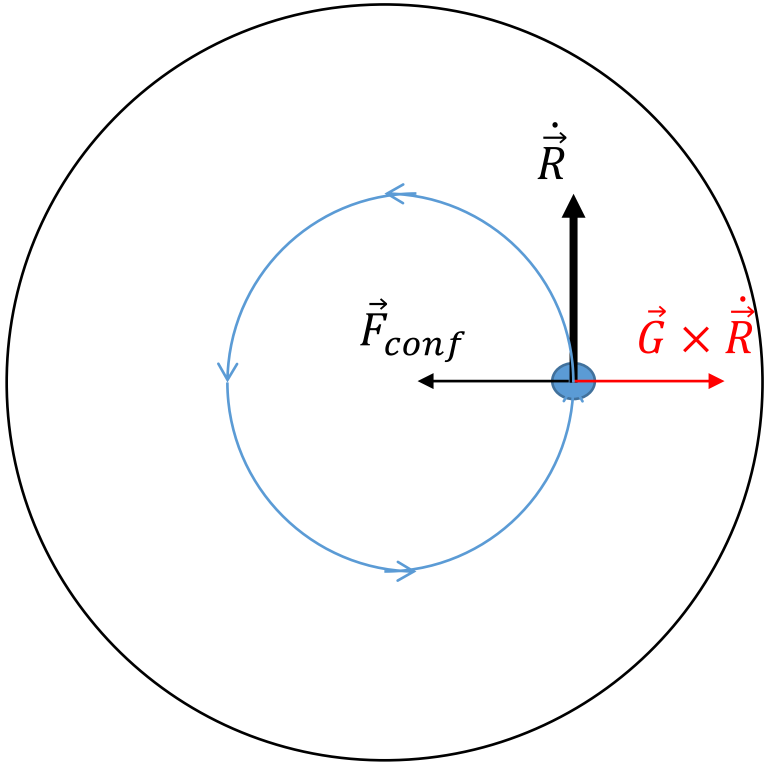
$$\vec{F}_T = -\mu_0 M_S \int (\vec{m} \times \vec{T}) . \frac{\partial \vec{m}}{\partial \vec{R}} d^2 r$$

$$\vec{F}_{ext} = -\frac{\delta E}{\partial \vec{R}}$$

- The gyrotropic force evidences the role of topology on the dynamics. Only pertinent for $\pi_2(\mathbb{S}^2)$
- The dissipation describes the energy loss ($P = -\vec{F}_{diss} . \vec{v} \propto -|\vec{v}|^2 < 0$)

Dynamics of topological textures - Vortex core

$$\vec{G} \times \dot{\vec{R}} - \alpha D \dot{\vec{R}} + \vec{F}_{conf} = \vec{0}$$



Gyrotropic force: $n_{\pi_2(S^2)} = \frac{1}{2}$ so $\vec{G} = -\frac{\mu_0 M_S t}{\gamma_0} 2\pi \vec{z} = G\vec{z}$

Dissipation: For an isotropic core $D_{xx} = D_{yy} = D$

The vortex core is centered at equilibrium due to the dipolar couplings, so the confinement force can be given by $\vec{F}_{conf} = -\kappa \vec{R}$ with $\kappa \propto \mu_0 M_S^2$

The trajectory is a circle (or a damped spiral)

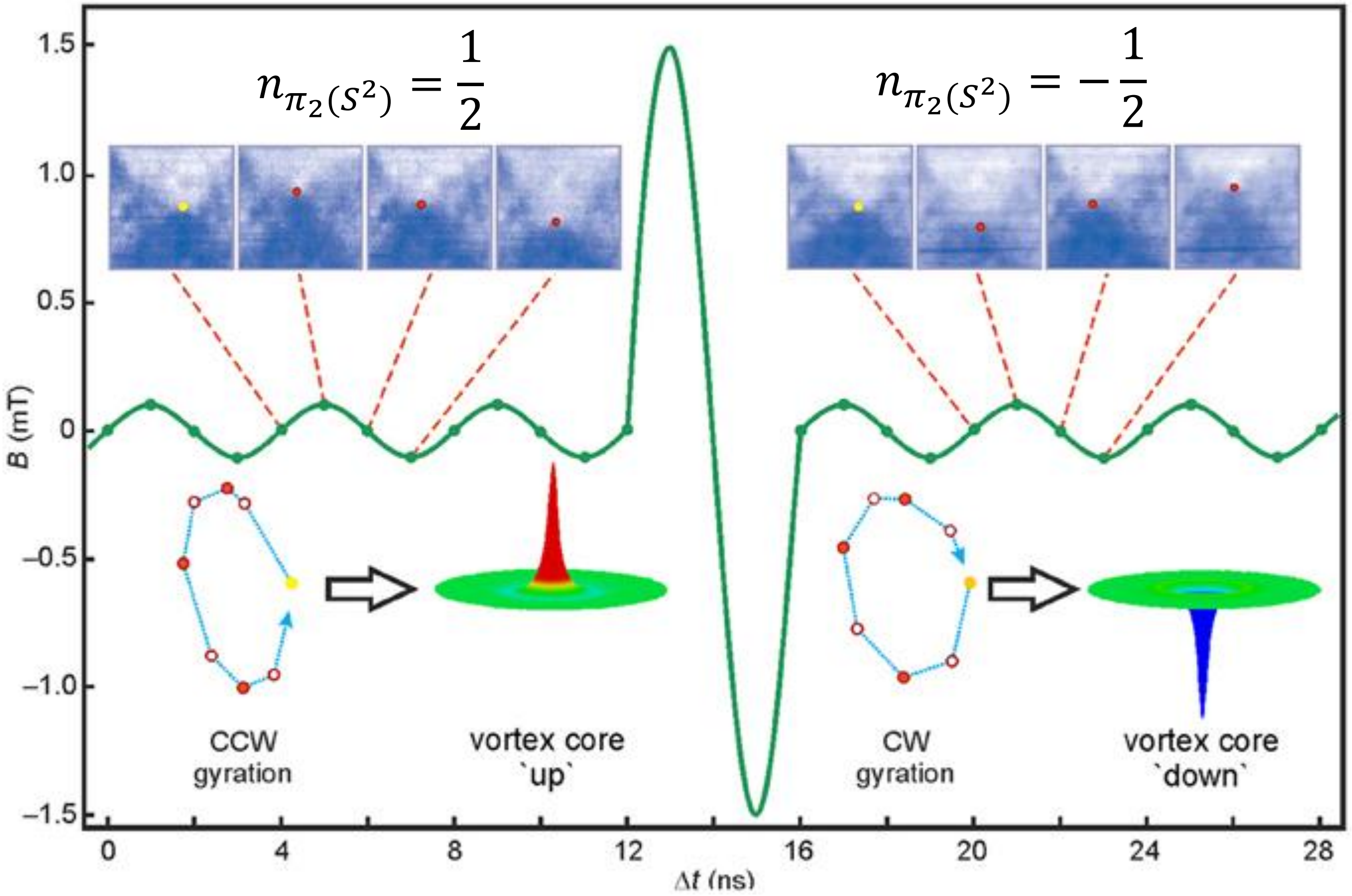
Undamped motion :

$\begin{cases} -G\dot{Y} - \kappa X = 0 \\ G\dot{X} - \kappa Y = 0 \end{cases}$ leads to $\begin{cases} \ddot{X} + \omega^2 X = 0 \\ \ddot{Y} + \omega^2 Y = 0 \end{cases}$

If offset from the center, the vortex core rotates (precesses) around the dot center at gyration frequency $\omega = \kappa / G$

Dynamics of topological textures
- Vortex core

$$\vec{G} \times \dot{\vec{R}} - \alpha D \dot{\vec{R}} + \vec{F}_{conf} = \vec{0}$$

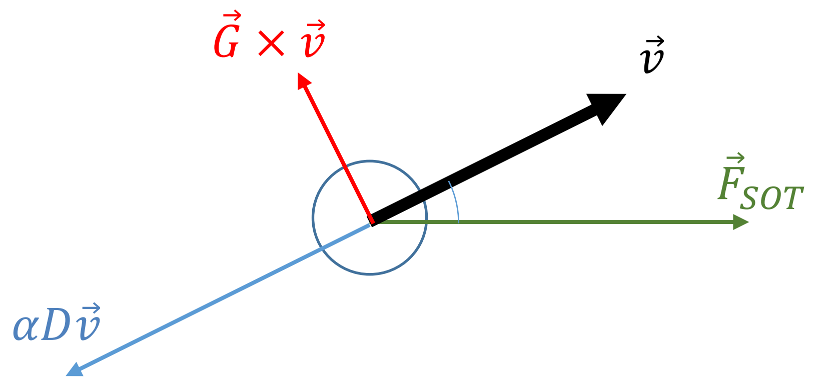


Switching of the vortex core by exciting the gyromode

[Van Waeyenberge et al. Nature 444, 461 (2006)]

Dynamics of topological textures - Skyrmion

$$\vec{G} \times \vec{v} - \alpha \overline{\overline{D}} \vec{v} + \vec{F}_T = \vec{0}$$



Gyrotropic force: $n_{\pi_2(S^2)} = 1$ so $\vec{G} = -\frac{\mu_0 M_S t}{\gamma_0} 4\pi \, \vec{z} = G\vec{z}$

Dissipation: For an isotropic skyrmion $D_{xx} = D_{yy} = D$

SOT Force: $\vec{F}_{SOT} \propto j\theta_H \cos\phi \vec{x}$

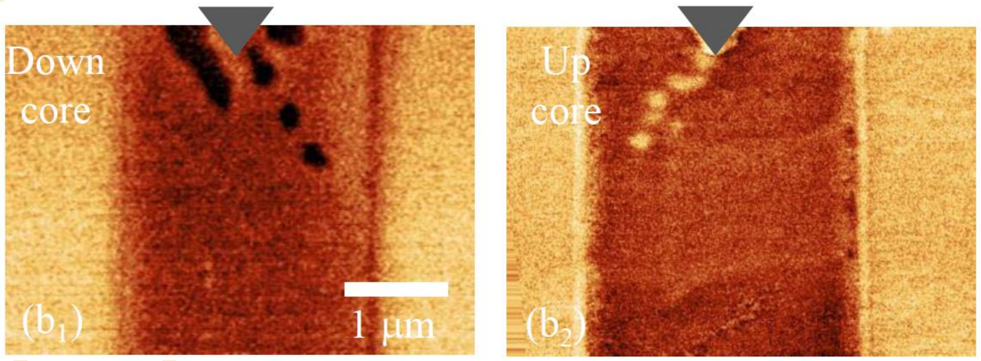
The motion is not along the current direction (skyrmion Hall effect):

Velocity $|v| = \left|\frac{F_{SOT}}{G}\right| \frac{1}{\sqrt{1+\rho^2}}$

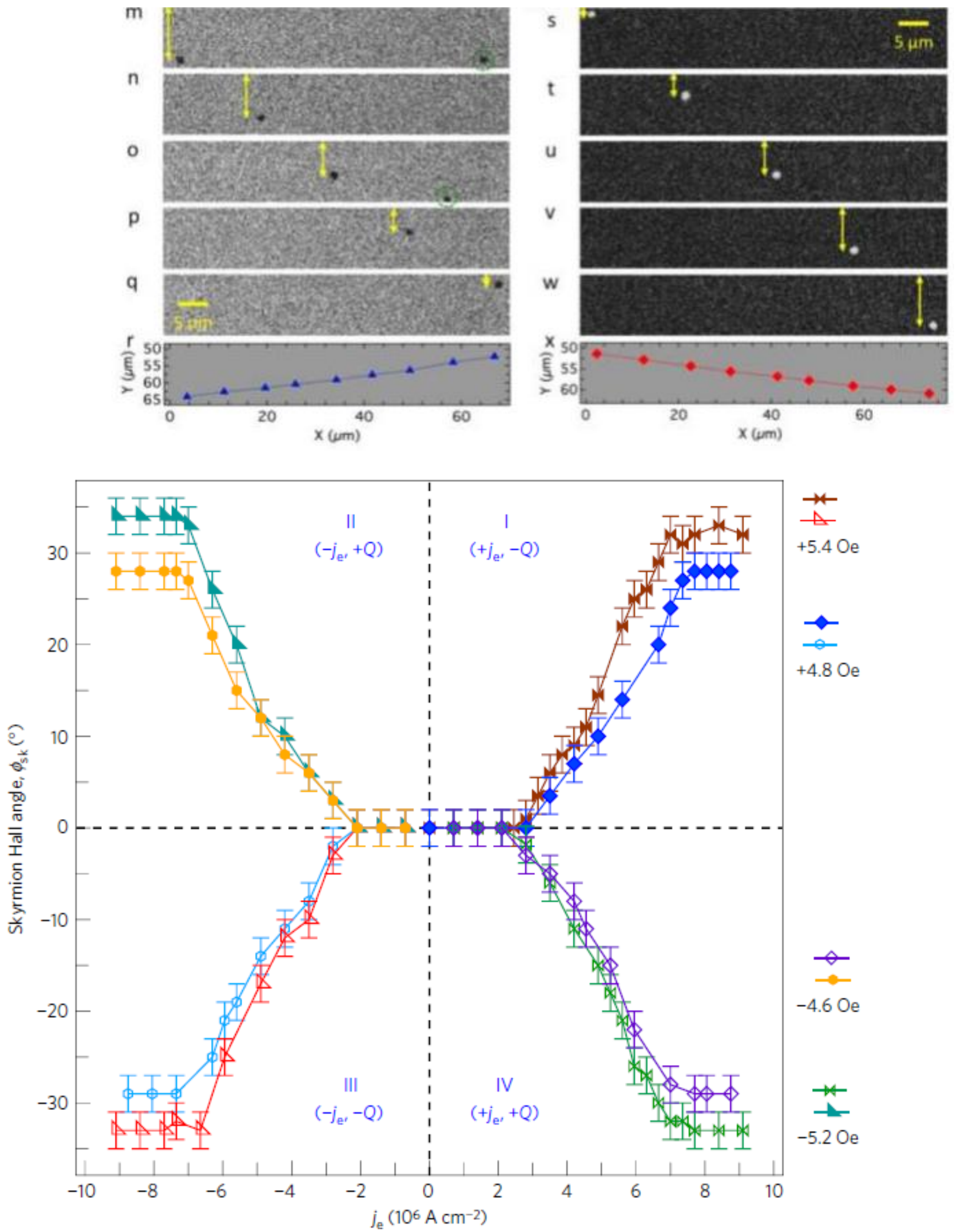
Angle $\rho = \frac{v_y}{v_x} = G/\alpha D$

Deflection depends on the sign of the gyrovector.
It can be reversed by switching the core polarization or by switching the winding number (skyrmion -> antiskyrmion)

Dynamics of topological textures - Skyrmion



Deflection of skyrmions in Pt/Co/Au based system.
[Mallick et al. Phys. Rev. Appl. 2022]



Deflection of skyrmions in Ta/CoFeB/TaOx system.
[Jiang et al. *N. Phys. 2017*]

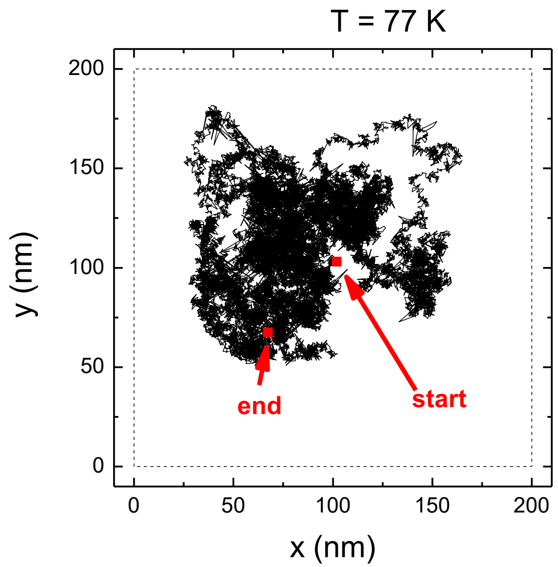
Dynamics of topological textures
- Skyrmion random walk

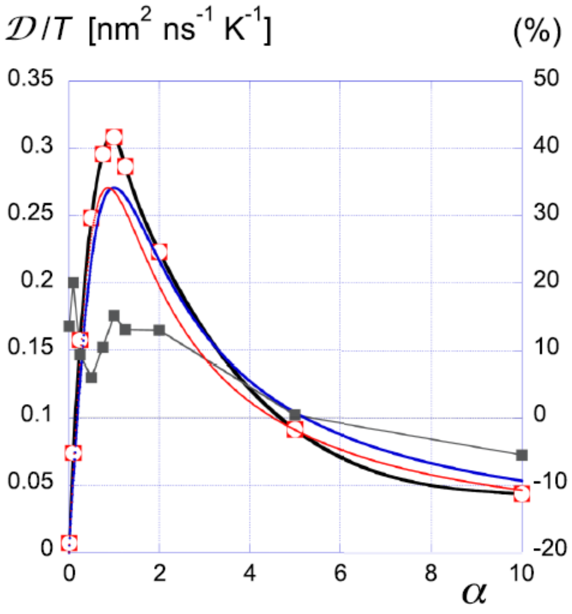
$$\vec{G} \times \vec{v} - \alpha D \vec{v} + \vec{F}_{thermal} = \vec{0}$$

The skyrmion is moved by a random force, due to thermal fluctuation.

For an isotropic skyrmion $\langle X^2(t) \rangle = \langle Y^2(t) \rangle = 2\mathcal{D}t$ with $\mathcal{D}$ the diffusion constant.

$$\mathcal{D} = k_B T \frac{\alpha D}{G^2 + (\alpha D)^2}$$





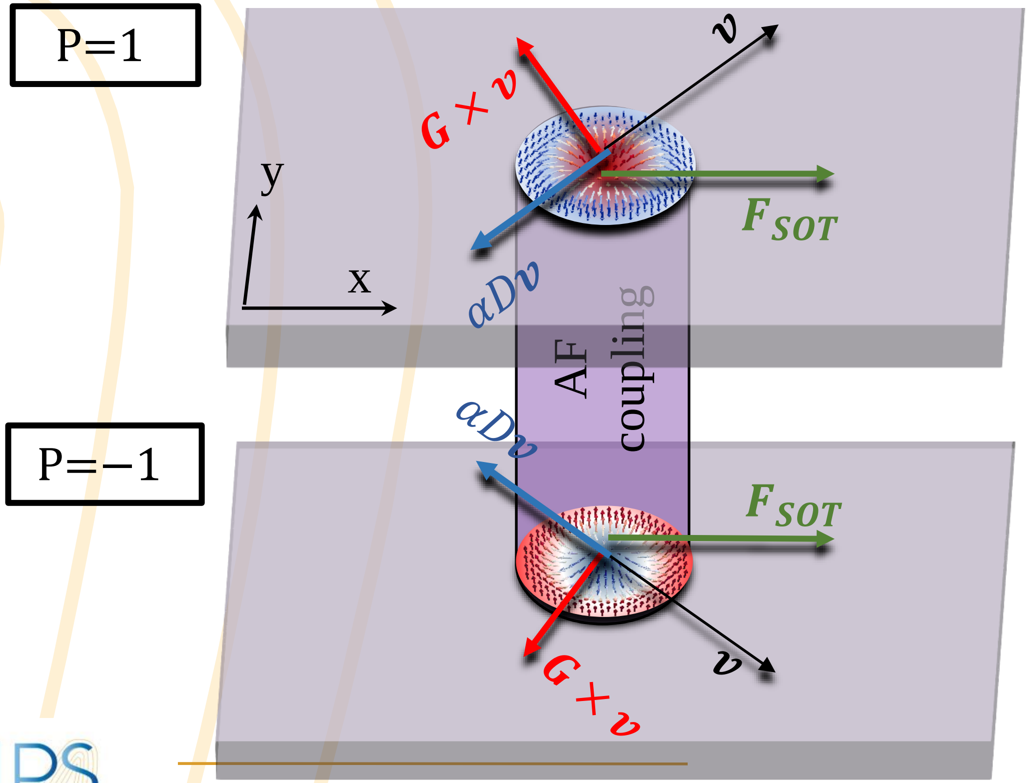
Consequence: skyrmions diffuse less due to their topology
[Miltat et al. Phys. Rev. B 2019]

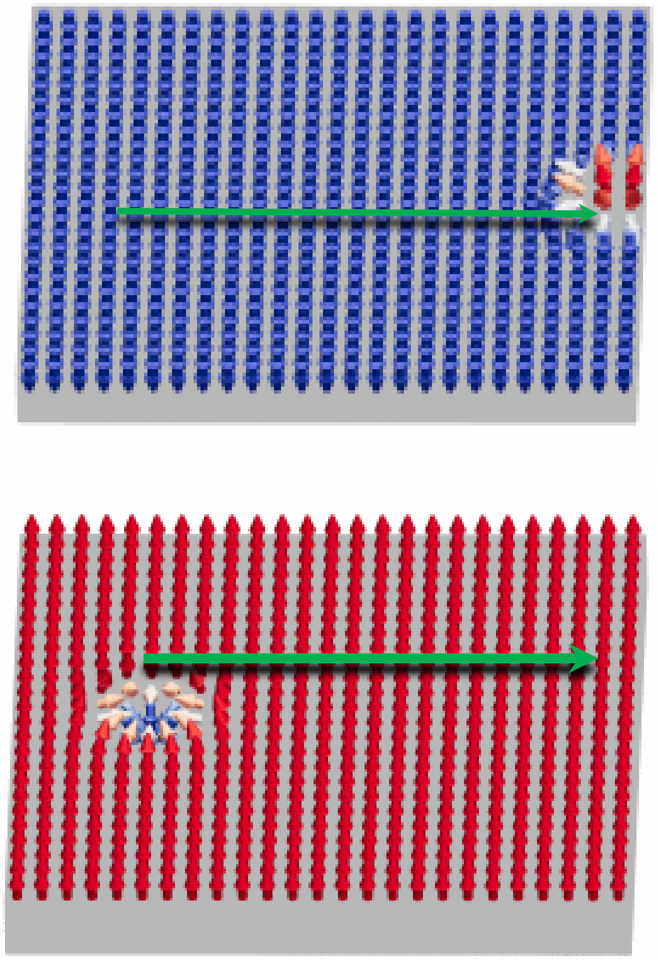
Dynamics of topological textures - Antiferromagnetic systems

Coupling two skyrmions with opposite core polarity

$$\boldsymbol{G} \times \boldsymbol{v} - \alpha D \boldsymbol{v} + \boldsymbol{F}_{SOT} = 0$$

$$\boldsymbol{G} = \frac{M_S t}{\gamma_0} 4\pi P$$

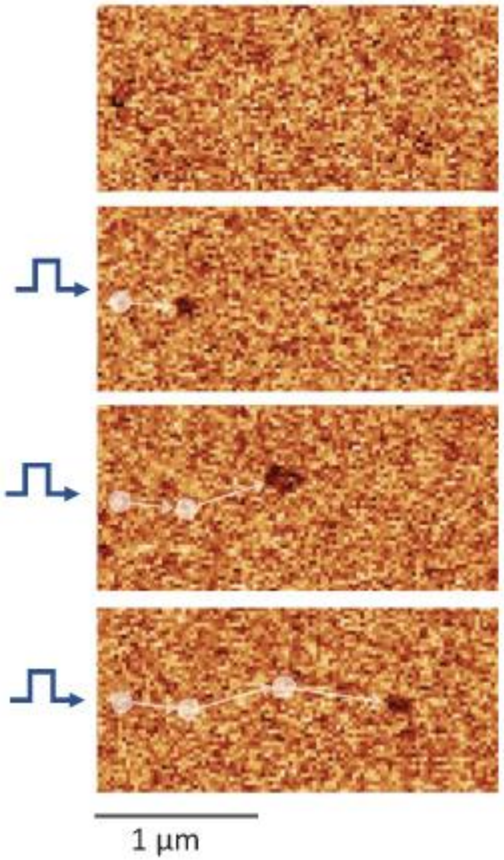


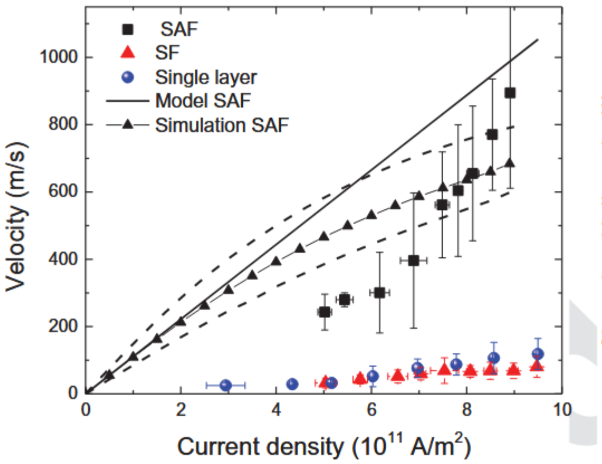


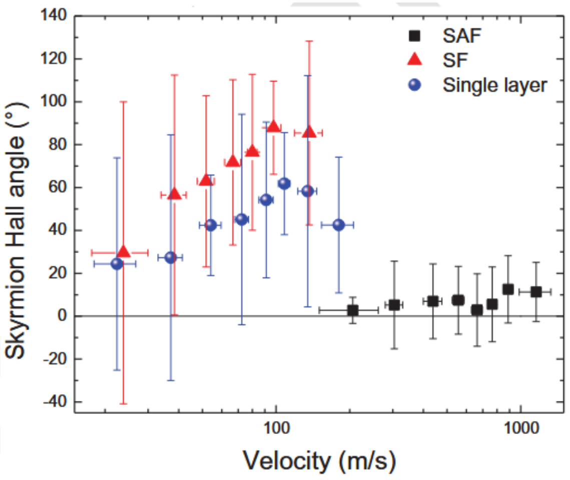
[Panigrahy et al. Phys Rev. B 2022]

Dynamics of topological textures
- Antiferromagnetic systems

Skyrmions in synthetic antiferromagnets:
2 AF coupled Co layers





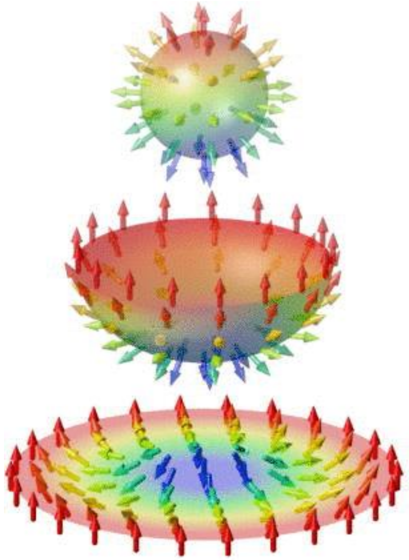


- No gyrotropic deflection
- Increased velocity

V.T. Pham et al.
Science 384, 6693 (2024)

Conclusion

- Topology in magnetic textures is particularly relevant for 0D textures ($\pi_2(\mathbb{S}^2)$ homotopy group)
- Topological transition are complex and are dominated by the exchange energy
- Topology of 0D textures has important consequences on the dynamics (gyrotropic effects)



Main references:

E. Feldtkeller, *Continuous and singular micromagnetic configurations* Z. Angew. Phys. 19, 530 (1965)

A.A. Thiele, *Steady-state motion of magnetic domains*, Phys. Rev. Lett. 30, 230 (1973)

A.A. Thiele, *Applications of the gyrocoupling vector and dissipation dyadic in the dynamics of magnetic domains*, J. Appl. Phys. 45, 377 (1974)

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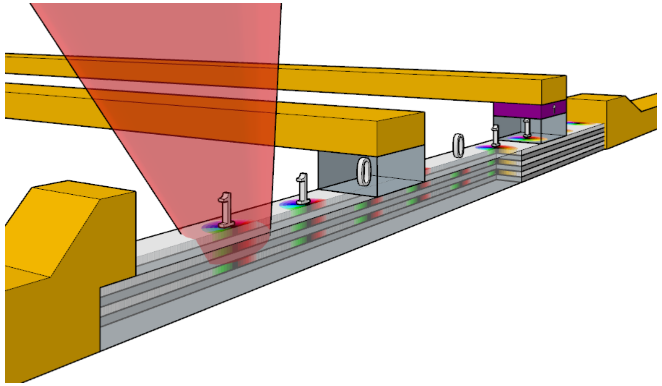

Main review paper:

H.B. Braun, *Topological effects in nanomagnetism: from superparamagnetism to chiral quantum solitons*, Adv. Phys. 61, 1 (2012)

A. Thiaville, J. Miltat and S. Rohart *Magnetism and topology* in *Magnetic skyrmions and their applications* Elsevier (2021)