Topological magnetic textures

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Introduction: magnetic textures

Ferromagnets can be composed of single magnetic domains or complex spin textures.

Textured magnetic ground state
- Induced by crystal microstructure
- Induced by micromagnetic energy balance

Metastable excitations => solitonic textures
- Vortex cores
- Magnetic domain walls
- Skyrmions
- ...

Size from few nanometers (vortex core, out-of-plane domain walls) to micrometers (in-plane domain walls)

Stripes stabilized by DMI + dipolar interaction in magnetic multilayer

Complex spin texture in PdNi nanostructure stabilized by strain relief induced anisotropy and dipolar interactions

Chaulieu, SR PRB 2011

Magnetic vortex in NiFe disc

Magnetic domain wall in NiFe stripe
Introduction: magnetic textures

Applications sin spintronics devices

Current induced domain wall motion

-> Race track memories (shift register) and logics

Spin-polarized current through the magnetic material drives excitation (domain wall motion, vortex core precession)

Vortex core dynamics

-> Narrow linewidth oscillators (sub GHz)

... and future concepts using new textures for logics, neuromorphic and probabilistic computing, cryptography...
Introduction: micromagnetic framework

**Magnetocrystalline anisotropy energy**

\[ E_{MC} = K (\vec{m} \cdot \vec{e})^2 \]

(simplest form, may be more complicated) reflects the crystal symmetry

**Exchange energy**

\[ E_{ex} = -JS_i \cdot S_j = A(\nabla \theta)^2 \]

**Dipolar energy**

\[ E_D = -\frac{\mu_0}{4\pi} \left( \frac{3(\vec{m}_i \cdot \vec{r}_j)}{r_j^5} - \frac{\vec{m}_i}{r_j^3} \right) \cdot \vec{m}_j \]

For practical use:
shape anisotropy

\[ E_D = -\frac{1}{2} \mu_0 \vec{M} \cdot \vec{H}_d \]

\[ = -\frac{1}{2} \mu_0 \vec{M} \cdot [N] \vec{M} \]

**Dyadoshinskii Moiya interaction**

\[ E_{DMI} = -d_{ij} \cdot (\vec{S}_i \times \vec{S}_j) \]

Requires absence of inversion symmetry (lattice or interface)

**Interface anisotropy**

\[ E_{interf.anis} = -\frac{K_S}{t} (\vec{m} \cdot \vec{z}) \]

**Zeeman energy**

\[ E_Z = -\mu_0 \vec{M} \cdot \vec{H} \]
Introduction: micromagnetic framework

- **Bloch wall**
  - Anisotropy vs. Exchange
  - $E = A\left(d\theta/dx\right)^2 + K \sin^2 \theta$
  - Length scale: $\delta_B = \sqrt{A/K}$
  - Bloch wall energy: $\sigma_B = 4\sqrt{AK}$

- **Exchange length**
  - Dipolar coupling vs. Exchange
  - $\Lambda = \frac{2A}{\mu_0M_s^2} \approx 2.6\Lambda$
  - Typical value: 5-10 nm

- **Dzyalinskii modulation**
  - DMI vs. Exchange
  - $\xi = 2A/D$
  - Length scale: $\xi = 2A/D$
  - Modulation period: $2\pi\xi$

**Quality factor**

- $Q = \frac{2K}{\mu_0M_s^2} = \left(\Lambda/\delta\right)^2$
  - $Q > 1$: hard
  - $Q << 1$: soft
Why do we care about topology in magnetism? Do we have properties that are directly related to topology?

- Stability
- Dynamics
Texture stability

We consider a soliton-like texture (domain wall, skyrmion, vortex)

-> How can we transit towards a state with a different topology?
-> How to relate energy with topology?

Domain wall in a stripe

Topology is held by boundary condition
Stability requires to take the domain wall out of the stripe
=> Nothing interesting related to topology

360° domain wall

$\pi_1(S^2)$ trivial topology
Stability is not related to topology but to specific energy terms (DMI, dipolar repulsion)

Skyrmion, magnetic vortex core

$\pi_2(S^2)$ non-trivial topology
Skyrmion collapse or vortex core reversal goes through a topological defect ($S^1$ vortex or $S^2$ Bloch point)
Skyrmion

- Chiral nanobubble

Néel Skyrmion (stabilized by interface DMI)

Bloch Skyrmion (stabilized by volume DMI in B20 crystals)

Order parameter space mapping:
Non trivial topology

\[ S = \frac{1}{4\pi} \int \vec{m} \left( \frac{\partial \vec{m}}{\partial x} \times \frac{\partial \vec{m}}{\partial y} \right) d^2r = 1 \]

\( \Rightarrow \)Impossible continuous transition toward the ferromagnetic phase (\( S = 0 \))
Skyrmions

First images using TEM Lorentz imaging in Fe$_{0.5}$Co$_{0.5}$Si


Interfaces stabilized skyrmions
Observed by SP-STM
Ir(111)/Fe(1ML)/Pd(1ML)

N. Romming et al. PRL 114, 177203 (2015)
Skyrmions: energy stabilization

Dipolar coupling: flux closure between core and surrounding

Transition: domain wall energy \( \sigma = 4\sqrt{AK} - \pi D \)
+ correction terms at small size

Rohart and Thiaville
Magnetic Vortex

Soft magnetic disc (no anisotropy): need to minimize dipolar energy

- \( \text{div } \vec{M} = 0 \)
- \( \vec{M} \cdot \hat{n} = 0 \)

But exchange energy divergence at the center

\[ \Rightarrow \text{Magnetization turns perpendicular over a distance } \Lambda = \frac{2A/\mu_0 M_S^2}{\mu_0} \]

\( \Rightarrow \text{Vortex core} \)

Beware: magnetic vortex leaves on \( \mathbb{S}^2 \).
It is not a XY (or \( \mathbb{S}^1 \)) vortex (no topological defect at the center).
In topology, it can be referred to as a *meron*.
Magnetic Vortex: topology

Homotopy group: $\pi_2(S^2)$

Sphere is covered once: $|n| = \frac{1}{2}$

Topology depends on the vortex core orientation $p$ and vorticity $W$ (topology of the periphery)

$$n_{\pi_2(S^2)} = pW = \pm \frac{1}{2}$$
Stability of topological textures

- Collapse of skyrmions and vortex core switching require a change in topology

**Skyrmion collapse**
- Core orientation
- Boundary condition at infinity (skyrmion surrounding)

**Vortex core switching**
- Core orientation
- Fixed boundary condition at nanostructure edge

=> Requires the injection of a magnetic defect ($S^1$ vortex or $S^2$ Bloch point)
Stabilization of topological textures
skyrmion colapse

Finite collapse energy can be evidenced in simulations at finite temperatures

⇒ Example of a skyrmion in a Co monolayer on Pt(111)
Simulation at the atomic scale

\[ t_{\text{survival}} = \exp \left( -\frac{t}{\tau} \right) \text{ and } \tau = \tau_0 \exp \frac{\Delta E}{k_B T} \]
\[ \tau = 0.2 \text{ ns, } \Delta = 27 \text{ meV} \]

Stabilization of topological textures
skyrmion collapse

The topological problem doesn’t exist at the atomic scale

Collapse path calculation
Nuged elastic band micromagnetics

Initial state (skyrmion)
\(n_{\pi_2(S^2)} = 1\)

Saddle point
\(n_{\pi_2(S^2)} \sim 0\)

Stabilization of topological textures
skyrmion colapse

Can we understand the stability from micromagnetics arguments?

**Skyrmion energy**

=> Calculation with micromagnetic framework

**S**

Micromagnetic energy

\[ E = 2\pi t \iint \left\{ A \left[ \left( \frac{d\theta}{dr} \right)^2 + \frac{\sin^2 \theta}{r^2} \right] - D \left[ \frac{d\theta}{dr} + \frac{\cos \theta \sin \theta}{r} \right] + K \sin^2 \theta - \mu_0 H_d \cdot \vec{M} \right\} r \, dr \]

\[ \sim \propto \text{skyrmion radius at large radius} \]

\[ \sim 4\pi At n_{\pi_2(S^2)} \] at small sizes

**Saddle point**

⇒ Only a few atoms in the skyrmion core

Prevent from using the micromagnetic hypothesis

Colapse saddle point configuration: energy is dominated by the exchange whose energy cannot vanish due to topology

\[ E_{saddle} \approx 4\pi At |n_{\pi_2(S^2)}| = 4\pi At \]

1 Belavin and Poliakov JETP Lett 22, 245 (1975)

Micromagnetics can estimate the energies with a good accuracy. For the energy barrier, the errors are propagating and the accuracy is poor.
Stabilization of topological textures
skyrmion colapse in thick samples

Collapse occurs via a Bloch (true 0D defect) point rather than a vortex (1D diffect in a thick sample)

From *Topological defect-mediated skyrmion annihilation in three dimensions*

See also Milde et al. *Unwinding of a skyrmion lattice by magnetic monopoles*
Science 340, 1076 (2013)
Stabilization of topological textures
vortex core reversal

Switching the vortex core modifies the topology from $S = \frac{1}{2}$ to $S = -\frac{1}{2}$.

Thick problem ($t > \Lambda = \sqrt{2A/\mu_0M_S^2}$):
magnetization is not constant along the $z$ direction.

Vortex core switching is not homogeneous: nucleation of a Bloch point.

Normal vortex state: magnetization is perpendicular to minimize exchange energy

Vortex with Bloch point: magnetic moment are all almost in plane, mean magnetization is zero at the core

Fig. 2. MFM image of an array of permalloy dots 1 \( \mu \)m in diameter and 50 nm thick. Shinjo et al. Science 289, 930 (2000)

Thiaville et al. PRB 67, 094410 (2003)
Images from R. Dittrich http://magnet.atp.tuwien.ac.at/gallery/bloch_point/index.html
Stabilization of topological textures
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Dynamics: basis of magnetization dynamics

Ferromagnetic dynamics

Landau-Lifshitz-Gilbert (LLG) equation:

\[
\frac{\partial m}{\partial t} = -\gamma_0 m \times H_{\text{eff}} + \alpha m \times \frac{\partial m}{\partial t} + \tau_{\text{STT}}
\]

Precession   Damping   Spin transfer torque


Integration over space: texture dynamics

Thiele equation

\[
G \times \nu - \alpha D \nu + F_{\text{STT}} = 0
\]

Thiele equation

Integrated over the whole space assuming no deformation


Dynamics of topological textures
Thiele equation

\[ \mathbf{\hat{G}} \times \mathbf{\hat{v}} - \alpha \mathbf{\overline{D}} \mathbf{\hat{v}} + \mathbf{\hat{F}}_{\text{ext}} + \mathbf{\hat{F}}_{\text{STT}} = \mathbf{0} \]

Gyrotropic deflection  Dissipation  External potential  Current induced force

\[
\mathbf{\hat{G}} = -\frac{\mu_0 M_s t}{\gamma_0} 4\pi n_{\pi_2(S^2)} \mathbf{\hat{Z}}
\]

\[
D_{ij} = \frac{\mu_0 M_s t}{\gamma_0} \int \int \left( \frac{\partial \mathbf{\hat{m}}_0}{\partial i} \cdot \frac{\partial \mathbf{\hat{m}}_0}{\partial j} \right) d^2r
\]

\[
\mathbf{\hat{F}}_T = -\mu_0 M_s \int (\mathbf{\hat{m}} \times \mathbf{\hat{T}}) \cdot \frac{\partial \mathbf{\hat{m}}}{\partial \mathbf{\hat{R}}} d^2r
\]

\[
\mathbf{\hat{F}}_{\text{ext}} = -\frac{\delta E}{\partial \mathbf{\hat{R}}}
\]

• The gyrotropic force evidences the role of topology on the dynamics. Only pertinent for \( \pi_2(S^2) \)

• The dissipation describes the energy loss (\( P = -\mathbf{\hat{F}}_{\text{diss.}} \cdot \mathbf{\hat{v}} \propto -|\mathbf{\hat{v}}|^2 < 0 \)
Dynamics of topological textures  
- Vortex core

\[ \hat{G} \times \hat{R} - \alpha D \hat{R} + \hat{F}_{conf} = \vec{0} \]

**Gyrotropic force:** \( n_{\pi_2(S^2)} = \frac{1}{2} \) so \( \hat{G} = -\frac{\mu_0 M_S t}{\gamma_0} 2\pi \hat{Z} = G \hat{Z} \)

**Dissipation:** For an isotropic core \( D_{xx} = D_{yy} = D \)

The vortex core is centered at equilibrium due to the dipolar couplings, so the confinement force can be given by \( \hat{F}_{conf} = -\kappa \hat{R} \) with \( \kappa \propto \mu_0 M_S^2 \)

The trajectory is a circle (or a damped spiral)

**Undamped motion:**

\[
\begin{align*}
- G \dot{Y} - \kappa X &= 0 \\
G \dot{X} - \kappa Y &= 0
\end{align*}
\]

leads to

\[
\begin{align*}
\ddot{X} + \omega^2 X &= 0 \\
\ddot{Y} + \omega^2 Y &= 0
\end{align*}
\]

If offset from the center, the vortex core rotates (precesses) around the dot center at gyration frequency \( \omega = \kappa / G \)
Dynamics of topological textures
- Vortex core

\[ \dot{G} \times \dot{R} - \alpha D \dot{R} + \vec{F}_{conf} = \vec{0} \]

Switching of the vortex core by exciting the gyromode

\[ n_{\pi_2(S^2)} = \frac{1}{2} \]
\[ n_{\pi_2(S^2)} = -\frac{1}{2} \]

[Van Waeyenberge et al. Nature 444, 461 (2006)]
Dynamics of topological textures - Skyrmion

\[ \mathbf{G} \times \mathbf{v} - \alpha \mathbf{D} \mathbf{v} + \mathbf{F}_T = \mathbf{0} \]

Gyrotropic force: \( n_{\pi_2(S^2)} = 1 \) so \( \mathbf{G} = -\frac{\mu_0 M_s}{\gamma_0} 4\pi \mathbf{Z} = G \mathbf{Z} \)

Dissipation: For an isotropic skyrmion \( D_{xx} = D_{yy} = D \)

SOT Force: \( \mathbf{F}_{SOT} \propto j \theta_H \cos \phi \mathbf{x} \)

The motion is not along the current direction (skyrmion Hall effect):

Velocity \( |v| = \left| \frac{\mathbf{F}_{SOT}}{G} \right| \frac{1}{\sqrt{1+\rho^2}} \)

Angle \( \rho = \frac{v_y}{v_x} = G/\alpha D \)

Deflection depends on the sign of the gyrovector.
It can be reversed by switching the core polarization or by switching the winding number (skyrmion -> antiskyrmion)
Dynamics of topological textures - Skyrmion


Deflection of skyrmions in Ta/CoFeB/TaOx system. [Jiang et al. N. Phys. 2017]
Dynamics of topological textures
- Skyrmion random walk

\[ \vec{G} \times \vec{v} - \alpha D \vec{v} + \vec{F}_{\text{thermal}} = \vec{0} \]

The skyrmion is moved by a random force, due to thermal fluctuation.

For an isotropic skyrmion \( \langle X^2(t) \rangle = \langle Y^2(t) \rangle = 2\mathcal{D}t \) with \( \mathcal{D} \) the diffusion constant.

\[ \mathcal{D} = k_B T \frac{\alpha D}{G^2 + (\alpha D)^2} \]

Dynamics of topological textures - Antiferromagnetic systems

Coupling two skyrmions with opposite core polarity

\[ \mathbf{G} \times \mathbf{v} - \alpha D \mathbf{v} + F_{SOT} = 0 \]

\[ \mathbf{G} = \frac{M_s \gamma_0 t}{4\pi P} \]

[Panigrahy et al. Phys Rev. B 2022]
Dynamics of topological textures
- Antiferromagnetic systems

Skyrmions in synthetic antiferromagnets:
2 AF coupled Co layers

- No gyrotropic deflection
- Increased velocity

V.T. Pham et al.
Science 384, 6693 (2024)
Conclusion

- Topology in magnetic textures is particularly relevant for 0D textures ($\pi_2(S^2)$ homotopy group)
- Topological transition are complex and are dominated by the exchange energy
- Topology of 0D textures has important consequences on the dynamics (gyrotropic effects)

Main references:

E. Feldtkeller, *Continuous and singular micromagnetic configurations* Z. Angew. Phys. 19, 530 (1965)

Main review paper:
A. Thiaville, J. Miltat and S. Rohart *Magnetism and topology in Magnetic skyrmions and their applications* Elsevier (2021)